English Version

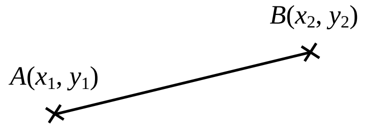
F.3 Coordinate Geometry

Revision Notes:

1. Distance formula 距離公式

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ n a rectangular coordinate plane is give by
直角坐標平面上任何兩點 $A(x_1, y_1)$和 $B(x_2, y_2)$之間的距離可表示成：

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



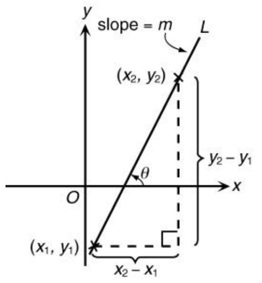
2. Slope and inclination **斜率與傾角**

The slope of the straight line L passing through (2, 1) and (3, 5)
通過 (2, 1) 和 (3, 5) 的直線 L 的斜率

$= \dfrac{5-1}{3-2}$ ◄ Substitute $x_1 = 2$, $y_1 = 1$, $x_2 = 3$ and $y_2 = 5$ into the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

$= \dfrac{4}{1}$

$= \underline{\underline{4}}$



∵ Slope of L L 的斜率= tan θ ◄ θ is the inclination of L. θ為 L 的傾角。

∴ $4 = \tan\theta$

$\theta = \underline{\underline{76.0°}}$ (cor. to 3 sig. fig.)

∴ The inclination of L is 76.0°. 直線 L 的傾角是 76.0°。

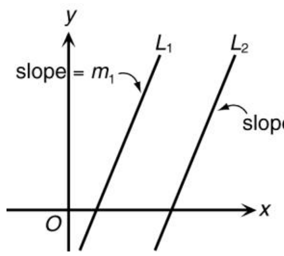
3. Parallel and perpendicular lines **平行線與垂直線**

Let m_1 and m_2 be the slopes of straight lines L_1 and L_2 respectively.
設 m_1 和 m_2 分別是直線 L_1 和 L_2 的斜率。

(a) Parallel lines 平行線

(i) If $L_1 // L_2$, then $m_1 = m_2$.
$L_1 // L_2$，則 $m_1 = m_2$。

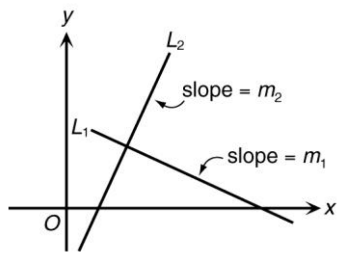
(ii) If $m_1 = m_2$, then $L_1 // L_2$.
若 $m_1 = m_2$，則 $L_1 // L_2$。



(b) Perpendicular lines 垂直線

(i) If $L_1 \perp L_2$, then $m_1 \times m_2 = -1$.
$L_1 \perp L_2$，則 $m_1 \times m_2 = -1$。

(ii) If $m_1 \times m_2 = -1$, then $L_1 \perp L_2$.
若 $m_1 \times m_2 = -1$，則 $L_1 \perp L_2$。

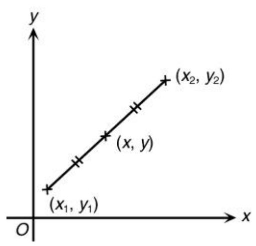


4. Mid-point formula

e.g. If $M(x, y)$ is the mid-point of the line segment joining $A(1, 6)$ and $B(3, 10)$, then
若 $M(x, y)$ 是連接 $A(1, 6)$ 和 $B(3, 10)$ 的線段的中點，則

$x = \dfrac{1+3}{2} = 2$ ◄ Substitute $x_1 = 1$ and $x_2 = 3$ into the formula $x = \dfrac{x_1 + x_2}{2}$.

$y = \dfrac{6+10}{2} = 8$ ◄ Substitute $y_1 = 6$ and $y_2 = 10$ into the formula $y = \dfrac{y_1 + y_2}{2}$.



5. Section formula

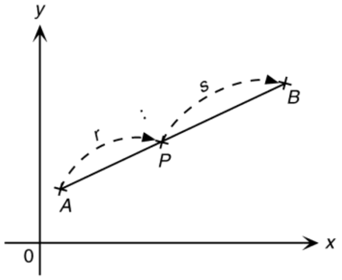
e.g. $P(x, y)$ is a point on the line segment joining $A(1, 1)$ and $B(4, 7)$. If $AP : PB = 1 : 2$, then

$$x = \frac{2(1)+1(4)}{1+2} = 2$$

◄ Substitute $x_1 = 1$, $x_2 = 4$, $r = 1$ and $s = 2$ into the formula $x = \frac{sx_1 + rx_2}{r + s}$.

$$y = \frac{2(1)+1(7)}{1+2} = 3$$

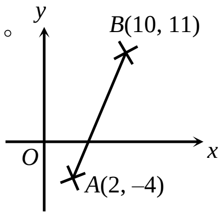
◄ Substitute $y_1 = 1$, $y_2 = 7$, $r = 1$ and $s = 2$ into the formula $y = \frac{sy_1 + ry_2}{r + s}$.



Worked examples:

e.g. 2.1 In the figure, the coordinates of A and B are (2, –4) and (10, 11) respectively. Find the distance between A and B.

圖中，A 和 B 的坐標分別是 (2, –4) 和 (10, 11)。求 A 和 B 之間的距離。



Solution

$AB = \sqrt{(10-2)^2 + [11-(-4)]^2}$ units 單位

$= \sqrt{8^2 + 15^2}$ units 單位

$= \sqrt{289}$ units 單位

= <u>17 units 單位</u>

e.g. 2.2 In the figure, $A(–3, 1)$, $B(4, 4)$ and $C(7, 1)$ are the vertices of $\triangle ABC$. Find the perimeter of $\triangle ABC$, correct to 3 significant figures.

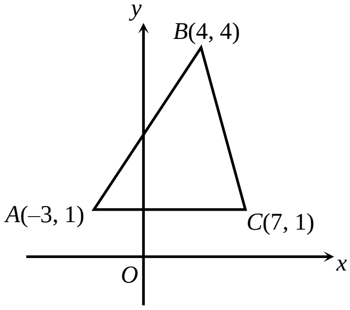
圖中，$A(–3, 1)$、$B(4, 4)$ 和 $C(7, 1)$ 是 $\triangle ABC$ 的頂點。求 $\triangle ABC$ 的周界，準確至三位有效數字。

Solution

$AC = [7 – (–3)]$ units 單位 = 10 units 單位

$AB = \sqrt{[4-(-3)]^2 + (4-1)^2}$ units 單位 $= \sqrt{58}$ units 單位

$BC = \sqrt{(7-4)^2 + (1-4)^2}$ units 單位 $= \sqrt{18}$ units 單位



Perimeter of $\triangle ABC$

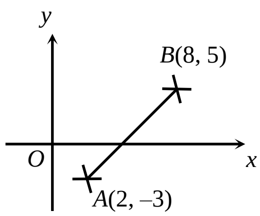
$= AC + AB + BC$

$= (10 + \sqrt{58} + \sqrt{18})$ units 單位

= <u>21.9 units 單位</u>, *cor. to 3 sig. fig. (準確至三位有效數字)*

e.g. 2.3 In the figure, the coordinates of A and B are (2, –3) and (8, 5) respectively. Find the slope of AB.

圖中，A 和 B 的坐標分別是 (2, –3) 和 (8, 5)。求 AB 的斜率。



Solution

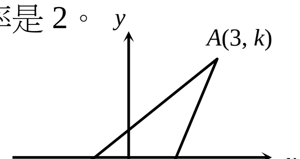
Slope of AB AB 的斜率

$= \frac{5-(-3)}{8-2}$

$= \underline{\underline{\frac{4}{3}}}$

e.g. 2.4 In the figure, $A(3, k)$, $B(1, –1)$ and $C(–2, –1)$ are the vertices of $\triangle ABC$. It is given that the slope of AB is 2.

圖中，$A(3, k)$、$B(1, –1)$ 和 $C(–2, –1)$ 是 $\triangle ABC$ 的頂點。已知 AB 的斜率是 2。



(a) Find the value of k. 求 k 的值。

(b) Find the slope of AC. 求 AC 的斜率。

Solution

(a) Slope of AB AB 的斜率= 2

$$\frac{-1-k}{1-3} = 2$$

$$k = \underline{\underline{3}}$$

(b) Slope of AC AC 的斜率

$$= \frac{-1-3}{-2-3}$$

$$= \underline{\underline{\frac{4}{5}}}$$

e.g. 2.5 Determine whether $P(-1, 7)$, $Q(3, -4)$ and $R(7, -15)$ are collinear.

判斷 $P(-1, 7)$、$Q(3, -4)$ 和 $R(7, -15)$ 是否共線。

Solution

Slope of PQ PQ 的斜率

$$= \frac{-4-7}{3-(-1)}$$

$$= -\frac{11}{4}$$

Slope of QR QR 的斜率

$$= \frac{-15-(-4)}{7-3}$$

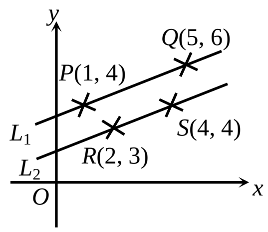
$$= -\frac{11}{4}$$

∵ Slope of PQ = slope of QR PQ 斜率 = QR 的斜率

∴ P, Q and R are collinear. P、Q 和 R 共線。

e.g. 2.6 In the figure, L_1 is a straight line passing through the points $P(1, 4)$ and $Q(5, 6)$ while L_2 is a straight line passing through the points $R(2, 3)$ and $S(4, 4)$. Show that $L_1 // L_2$.

Solution



Slope of L_1 L_1的斜率 $= \frac{6-4}{5-1} = \frac{1}{2}$

Slope of L_2 L_2的斜率$= \frac{4-3}{4-2} = \frac{1}{2}$

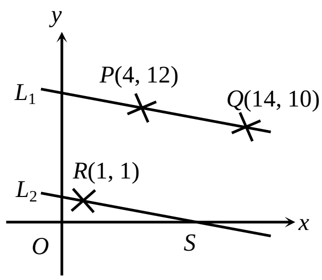
∵ Slope of L_1 = slope of L_2 L_1的斜率 = L_2的斜率

∴ $L_1 // L_2$

e.g. 2.7 In the figure, L_1 is a straight line passing through the points $P(4, 12)$ and $Q(14, 10)$. A straight line L_2 passes through the point $R(1, 1)$ and cuts the x-axis at S. If $L_1 // L_2$, find the coordinates of S.

圖中，L_1是一條通過 $P(4, 12)$ 和 $Q(14, 10)$ 的直線。L_2是一條通過 $R(1, 1)$ 的直線且與 x 軸相交於 S。若 $L_1 // L_2$，求 S 的坐標。

Solution



Let $(s, 0)$ be the coordinates of S. 設 S 的坐標為 $(s, 0)$。

∵ $L_1 // L_2$

∴Slope of L_1 = slope of L_2 L_1的的斜率= L_2的斜率

$$\frac{10-12}{14-4} = \frac{0-1}{s-1}$$

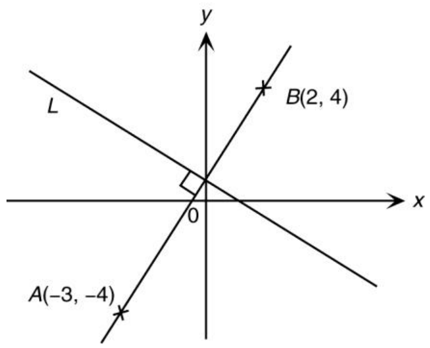
$$s = 6$$

∴ The coordinates of S are (6, 0). S 的坐標是 (6, 0)。

e.g. 2.8 In each of the following, the straight line L is perpendicular to AB. Find the slope of L.
在下列各題中，直線 L 垂直於 AB。求 L 的斜率。

Solution:

(a)



$$\text{Slope of } AB = \frac{4-(-4)}{2-(-3)} = \frac{8}{5}$$

$\because$ $L \perp AB$

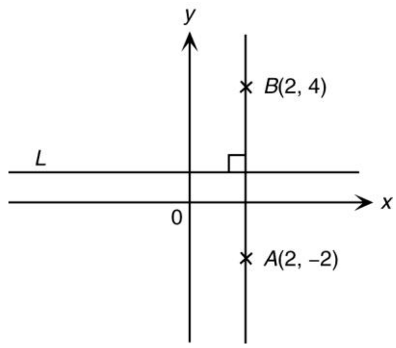
$\therefore$ Slope of L × slope of AB = –1

$$\text{Slope of } L \times \frac{8}{5} = -1$$

$$\text{Slope of } L = -1 \times \frac{5}{8}$$

$$= \underline{\underline{-\frac{5}{8}}}$$

(b)



$\because$ $L \perp AB$

and AB is a vertical line.

$\therefore$ L is a horizontal line.

$\therefore$ Slope of L = $\underline{\underline{0}}$

$\because$ $L \perp AB$

及 AB 是一條鉛垂線。

$\therefore$ L 是一條水平線。

$\therefore$ L 的斜率 = $\underline{\underline{0}}$

e.g. 2.9 (a) Given $A(2, -1)$, $B(6, 11)$, (x, y) is the mid-point of AB. Find the coordinates of M.
已知 $A(2, -1)$及 $B(6, 11)$ ，$M(x, y)$ 是 AB 的中點。求 M 的坐標。
(b) Given $A(1, 0)$, $M(3, 7)$, M is the mid-point of PQ. Find the coordinates of $Q(x, y)$.
已知 $A(1, 0)$及 $M(3, 7)$ ，M 是 PQ 的中點。求 $Q(x, y)$ 的坐標。

Solution:

(a) By the mid-point formula, we have 根據中點公式，可得：

$$x = \frac{(2)+(6)}{(2)} \qquad y = \frac{-1+11}{2}$$
$$= 4 \qquad\qquad = 5$$

$\therefore$ Coordinates of M M 的坐標 $= \underline{\underline{(4, 5)}}$

(b) By the mid-point formula, we have

$$(3) = \frac{(1)+x}{2} \quad \text{and} \quad (7) = \frac{(0)+y}{2}$$

$\therefore$ $x = 5$ and $y = 14$

$\therefore$ Coordinates of $Q = \underline{\underline{(5, 14)}}$

e.g. 2.9 Given $A(-6, 3)$, $B(6, -6)$, $P(x, y)$ is a point on the line segment AB such that $AP : PB = 2 : 1$.
Find the coordinates of P.
已知 $A(-6, 3)$及 $B(6, -6)$，(x, y) 是連接 $A(1, 1)$ 和 $B(4, 7)$ 的線段上的一點，其中 $AP : PB = 2 : 1$。求 P 的坐標。

Solution:

By the section formula of internal division, we have 根據內分點的截點公式，可得：

$$x = \frac{1(-6)+2(6)}{(2)+(1)} \qquad y = \frac{1(3)+2(-6)}{(2)+(1)}$$

$$= (2) \qquad\qquad = (-3)$$

$\therefore$ Coordinates of $P = \underline{\underline{(2, -3)}}$

Exercise

1. In each of the following, find the distance between the two given points.
(*Leave the answers in surd form if necessary*.)

(a) $A(4, -3)$, $B(-2, 5)$ **(b)** $C(-9, 1)$, $D(6, -7)$
(c) $P(-6, -1)$, $Q(-1, 2)$ **(d)** $R(4, 10)$, $S(8, 3)$

2. In each of the following, find the slope of the straight line passing through the two given points.

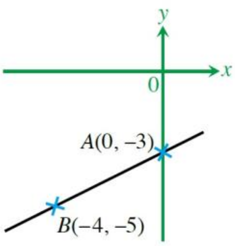
(a) $A(-3, -5)$, $B(0, 1)$ **(b)** $C(1, -7)$, $D(4, 8)$
(c) $P(2, -7)$, $Q(6, -4)$ **(d)** $R(-5, -3)$, $S(7, 11)$

3. In each of the following, determine whether the three given points are collinear.

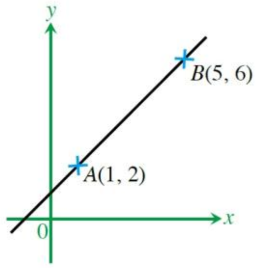
(a) $A(-5, -4)$, $B(1, -1)$, $C(7, 2)$ **(b)** $D(0, 5)$, $E(4, 3)$, $F(9, -1)$
(c) $P(-3, 6)$, $Q(5, 4)$, $R(9, 3)$ **(d)** $S(-4, -6)$, $T(2, -2)$, $U(11, 4)$

4. In each of the following, find the inclination of the straight line passing through A and B.
(*Give the answers correct to 3 significant figures if necessary*.)

(a)



(b)



5. If the slope of the straight line passing through the points $P(k, -1)$ and $Q(-4, 3k+2)$ is $-\frac{3}{2}$, find the value of k.

6. In each of the following, find the slope of a straight line parallel to the line segment PQ.

(a) $P(4, -2)$, $Q(7, 3)$ **(b)** $P(-5, 5)$, $Q(1, 4)$

7. In each of the following, find the slope of a straight line perpendicular to the line segment AB.

(a) $A(-1, -5)$, $B(3, 9)$ **(b)** $A(6, 1)$, $B(10, -2)$

8. In each of the following, show that $AB \parallel CD$.

(a) $A(1, 3)$, $B(7, 12)$, $C(-2, -6)$, $D(8, 9)$
(b) $A(-4, 8)$, $B(11, 2)$, $C(3, 5)$, $D(13, 1)$

9. In each of the following, show that $PQ \perp RS$.

(a) $P(-4, 6)$, $Q(5, 3)$, $R(-1, -9)$, $S(2, 0)$
(b) $P(-7, -5)$, $Q(-2, -1)$, $R(-8, 6)$, $S(4, -9)$

10. Consider the points $A(0, 4)$, $B(n-1, n)$, $C(2, -5)$ and $D(6, -3)$ on a rectangular coordinate plane. Find the value of n such that

(a) $AB \parallel CD$, **(b)** $AB \perp CD$.

11. In each of the following, find the coordinates of the mid-point of the line segment joining the two given points.

(a) $A(3, -7)$, $B(5, 9)$ **(b)** $C(-1, 6)$, $D(7, -8)$
(c) $P(-4, 3)$, $Q(-2, -11)$ **(d)** $R(-5, 2)$, $S(-9, 10)$

12. M is the mid-point of the line segment PQ. The coordinates of M and P are (6, –2) and (4, 5) respectively. Find the coordinates of Q.

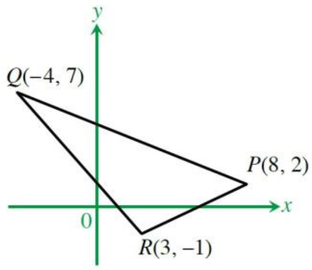
13. If $P(–3, –8)$ bisects the line segment joining the points $A(–7, p)$ and $B(q, 2)$, find the values of p and q.

14. In each of the following, P is a point lying on the line segment joining the points A and B. Find the coordinates of P.
(a) $A(4, 9)$, $B(7, –3)$; $AP : PB = 2 : 1$
(b) $A(3, –6)$, $B(10, 1)$; $AP : PB = 4 : 3$
(c) $A(–5, 8)$, $B(9, –13)$; $AP : PB = 5 : 2$
(d) $A(–4, 2)$, $B(6, 12)$; $AP : PB = 3 : 7$

15. P is a point lying on the line segment AB such that $AP : PB = 2 : 3$. The coordinates of B and P are (10, –5) and (1, 1) respectively. Find the coordinates of A.

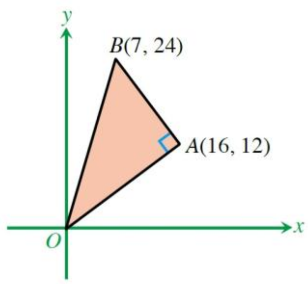
16. $P(–4, –2)$ is a point lying on the line segment joining the points $A(h, –11)$ and $B(–3, k)$ If $AP : PB = 3 : 1$, find the values of h and k.

17. In the figure, $P(8, 2)$, $Q(–4, 7)$ and $R(3, –1)$ are the vertices of a triangular field. A farmer wants to surround the field by fences of length 30 m. Is the length of fences long enough? Explain your answer.

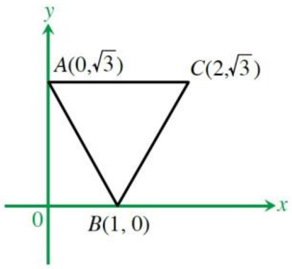


18. In the figure, O is the origin. The coordinates of A and B are (16, 12) and (7, 24) respectively. $\triangle OAB$ is right-angled at A.

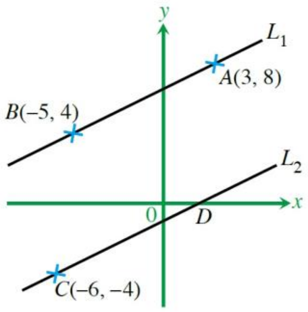
(a) Find the lengths of the line segments OA and AB.
(b) Find the area of $\triangle OAB$.



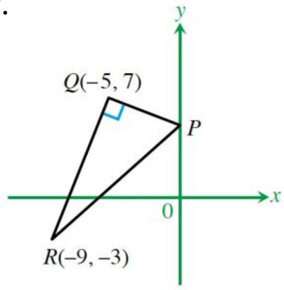
19. In the figure, $A(0, \sqrt{3})$, $B(1, 0)$ and $C(2, \sqrt{3})$ are the vertices of $\triangle ABC$. Is $\triangle ABC$ an equilateral triangle? Explain your answer.



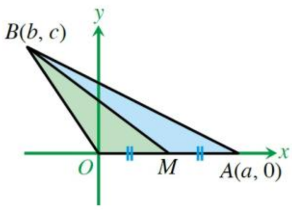
20. In the figure, L_1 is a straight line passing through the points $A(3, 8)$ and $B(–5, 4)$. A straight line L_2 passes through $C(–6, –4)$ cuts the x-axis at D. If $L_1 // L_2$, find the coordinates of D.



21. In the figure, $\triangle PQR$ is right-angled at Q. The coordinates of Q and R are (–5, 7) and (–9, –3) respectively. P is a point lying on the y-axis. Find the coordinates of P.

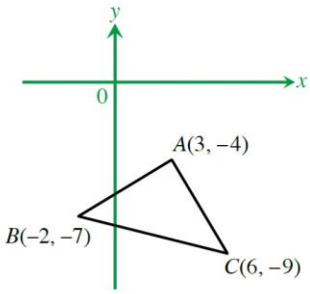


22. In the figure, $\triangle OAB$ has vertices $A(a, 0)$, $B(b, c)$ and $O(0, 0)$. Let M be the mid-point of OA. Prove that $\triangle ABM$ and $\triangle BOM$ have the same area.



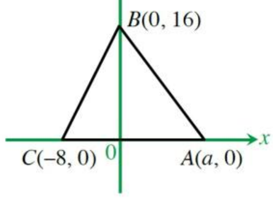
23. In the figure, $A(3, -4)$, $B(-2, -7)$ and $C(6, -9)$ are the vertices of $\triangle ABC$.

(a) Find the lengths of AB, AC and BC.
(b) Show that $\triangle ABC$ is a right-angled triangle.



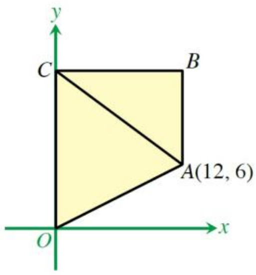
24. In the figure, $A(a, 0)$, $B(0, 16)$ and $C(-8, 0)$ are the vertices of an isosceles triangle ABC with $AB = AC$.

(a) Find the value of a.
(b) Find the perimeter of $\triangle ABC$, correct to 3 significant figures.

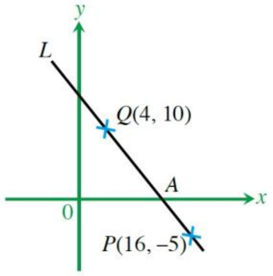


25. In the figure, $OABC$ is a quadrilateral. C is a point lying on the positive y-axis and the coordinates of A are (12, 6). The lengths of AC and OC are equal.

(a) Find the coordinates of C.
(b) If AB is vertical and BC is horizontal, find the area of $OABC$.

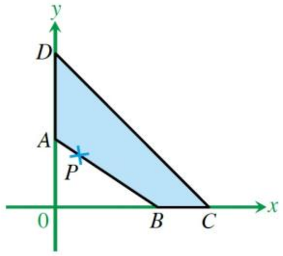


26. In the figure, L is a straight line passing through $P(16, -5)$ and $Q(4, 10)$. L cuts the x-axis at A. Find the coordinates of A.



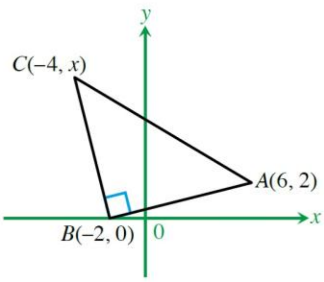
27. In the figure, B and C are points lying on the x-axis. A and D are points lying on the y-axis. P is a point lying on the line segment AB. The coordinates of B, C, D and P are $(12, 0)$, $(18, 0)$, $(0, 18)$ and $(3, 6)$ respectively.

(a) Find the coordinates of A.
(b) Find the area of the quadrilateral $ABCD$.



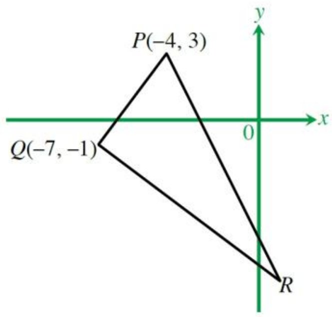
28. In the figure, $A(6, 2)$, $B(-2, 0)$ and $C(-4, x)$ are the vertices of a triangle ABC right-angled at B.

(a) Find the value of x.
(b) Find $\angle BAC$.



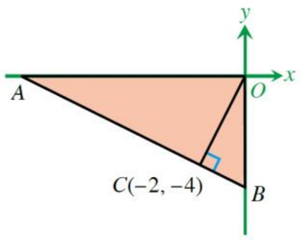
29. In the figure, the coordinates of P and Q are $(-4, 3)$ and $(-7, -1)$ respectively. Q is rotated anticlockwise about the origin through 90° to R.

(a) Write down the coordinates of R.
(b) Is $\triangle PQR$ a right-angled triangle? Explain your answer.

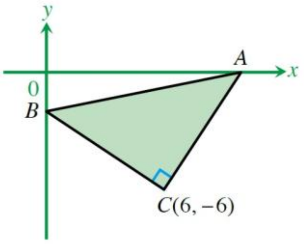


30. In the figure, A lies on the x-axis and B lies on the y-axis. $C(-2, -4)$ is a point lying on AB such that $OC \perp AB$.

(a) Find the coordinates of A and B.
(b) Find the area of $\triangle OAB$.



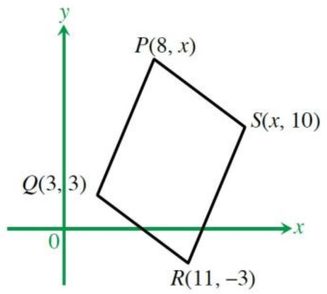
31. In the figure, A lies on the x-axis and B lies on the y-axis. $\triangle ABC$ is right-angled at C and the coordinates of C are $(6, -6)$. The slope of AC is $\frac{3}{2}$.



(a) Find the coordinates of A and B.

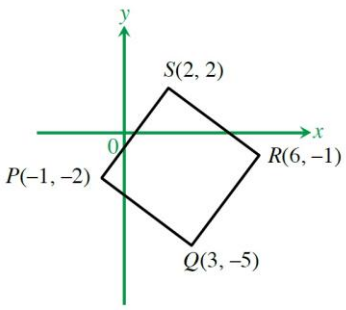
(b) Find the area of $\triangle ABC$.

32. In the figure, $P(8, x)$, $Q(3, 3)$, $R(11, -3)$ and $S(x, 10)$ are the vertices of a quadrilateral $PQRS$, where PS // QR.



(a) Find the value of x.
(b) Find the perimeter of $PQRS$, correct to 3 significant figures.

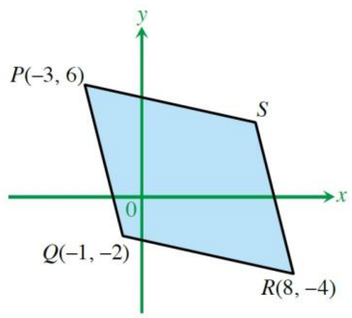
33. In the figure, $P(-1, -2)$, $Q(3, -5)$, $R(6, -1)$ and $S(2, 2)$ are the vertices of a quadrilateral $PQRS$.



(a) Find PQ, QR, RS and PS.
(b) Show that $PQRS$ is a square.

34. In the figure, $PQRS$ is a parallelogram. The coordinates of P, Q and R are $(-3, 6)$, $(-1, -2)$ and $(8, -4)$ respectively.

(a) Find the coordinates of S.
(b) $T(x, y)$ is a point lying on PQ.
(i) Express y in terms of x.
(ii) Find the coordinates of T if $ST \perp PQ$.
(iii) Find the area of $PQRS$.



35. Consider the points $A(3, 1)$ and $B(-2, 4)$. C is a point lying on the x-axis such that $AC = BC$.
(a) Find the coordinates of C.
(b) A student claims that C is the mid-point of AB. Do you agree? Explain your answer.

36. If $M(p, q)$ is the mid-point of the line segment joining the points $A(q, 7)$ and $B(1, p)$, find the values of p and q.

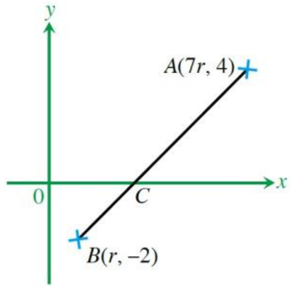
37. $P(-2, a)$ is a point lying on the line segment joining the points $Q(2b, 5)$ and $R(a, b)$. If $QP : PR = 2 : 5$, find the values of a and b.

38. Consider the points $A(-10, 3p)$ and $B(6, p)$. The line segment AB is produced to cut the x-axis at C.
(a) Find $AB : BC$.
(b) Let $p = 4$. Using the result of **(a)**, find the coordinates of C.

39. Consider the points $A(7, 1)$, $B(5, -1)$, $C(-3, -9)$ and $D(-2, -8)$. It is given that M is the mid-point of AC.
(a) Find the coordinates of M.
(b) Show that B, M and D are collinear.
(c) Find $BM : MD$.

40. The line segment AB cuts the x-axis and the y-axis at P and Q respectively. If the coordinates of A are $(-6, 6)$ and $AP : PQ : QB = 3 : 3 : 2$, find the coordinates of B, P and Q.

41. In the figure, the line segment joining the points $A(7r, 4)$ and $B(r, -2)$ cuts the x-axis at C.



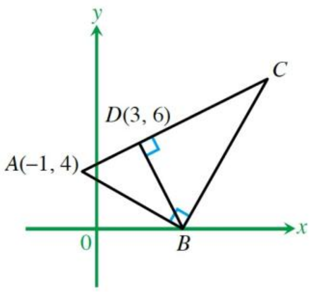
(a) Find $AC : CB$.

(b) The line segment AB is produced to cut the y-axis at D and let $r = 1$.

  (i) Find $DB : BC$.

  (ii) Find $AC : CB : BD$.

42. In the figure, $\triangle ABC$ is right-angled at B which lies on the x-axis. D is a point lying on AC such that $BD \perp AC$. The coordinates of A and D are $(-1, 4)$ and $(3, 6)$ respectively.

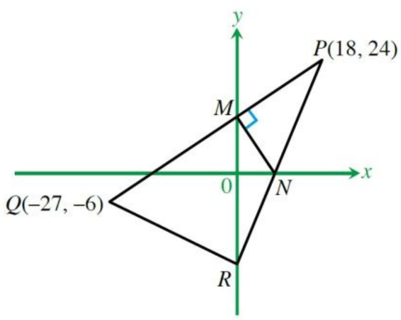


(a) Find the coordinates of B.

(b) Find the coordinates of C.

(c) Find $AD : DC$.

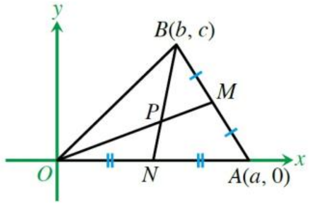
43. In the figure, the line segment joining $P(18, 24)$ and $Q(-27, -6)$ cuts the y-axis at M. N is a point lying on the x-axis such that $MN \perp PQ$. The line segment PN is produced to meet the y-axis at R.



(a) Find the coordinates of M and N.

(b) Find $PN : NR$.

44. In the figure, $\triangle OAB$ has vertices $A(a, 0)$, $B(b, c)$ and $O(0, 0)$. M and N are the mid-points of AB and OA respectively. OM and BN intersect at P.



(a) Let P_1 and P_2 be the points on BN and OM respectively such that $BP_1 : P_1N = OP_2 : P_2M = 2 : 1$. Find the coordinates of P_1 and P_2.

(b) Hence, prove that $OP : PM = BP : PN = 2 : 1$.

Multiple Choice Questions

45. Which of the following points is farthest from the origin?

A. (–1, 7) **B.** (2, 5) **C.** (4, –6) **D.** (0, 7)

46. A straight line cuts the x-axis and the y-axis at $A(p, 0)$ and $B(0, -p)$ respectively, where p is a positive constant. Find the slope of the straight line.

A. 1 **B.** –1 **C.** $2p$ **D.** $-2p$

47. Consider the points $A(3, 8)$ and $B(-2, -2)$. C is a point lying on the y-axis such that $AB \perp BC$. Find the coordinates of C.

A. (0, 3) **B.** (0, 0) **C.** (0, –1) **D.** (0, –3)

48. $M(4, -3)$ is the mid-point of the line segment AB. A and B are points lying on the x-axis and the y-axis respectively. Find the coordinates of A and B.

A. $A(8, 0)$, $B(0, -6)$ **B.** $A(0, 8)$, $B(-6, 0)$
C. $A(0, -6)$, $B(8, 0)$ **D.** $A(-6, 0)$, $B(0, 8)$

49. $P(5, -8)$, $Q(k, 4k)$ and $R(-10, 2)$ are collinear. Find $PQ : QR$.

A. 3 : 2
B. 2 : 3
C. k : 2
D. k : 3

DSE Type Questions

1. The coordinates of the points A and B are (–4, –2) and (3, 5) respectively. A is rotated anticlockwise about the origin O through 90° to A'. B is translated leftwards by 10 units to B'.

(a) Write down the coordinates of A' and B'.

(b) Prove that AB is perpendicular to $A'B'$.

2. The coordinates of the points P and Q are (–1, –4) and (1, 4) respectively. P is rotated anticlockwise about the origin O through 90° to P'. Q is translated downwards by 17 units to Q'.

(a) Write down the coordinates of P' and Q'.

(b) Prove that PQ is parallel to $P'Q'$.

3. The coordinates of the points C and D are (3, 6) and (2, 5) respectively. C is reflected along the y-axis to C'. D is rotated clockwise about the origin O through 90° to D'.

(a) Write down the coordinates of C' and D'.

(b) Prove that CD is perpendicular to $C'D'$.

DSE type MC (Optional)

1. Let O be the origin. If the coordinates of the points A and B are (20, 0) and (20, 15) respectively, then the x-coordinate of the circumcentre of $\triangle OAB$ is

A. 7.5. B. 10. C. 15. D. 20.

2. Let O be the origin. If the coordinates of the points M and N are (0, 18) and (–26, 18) respectively, then the y-coordinate of the circumcentre of $\triangle OMN$ is

A. –26. B. –13. C. 0. D. 9.

3. Let O be the origin. If the coordinates of the points P, Q and R are (9, 36), (41, 36) and (41, 11) respectively, then the y-coordinate of the circumcentre of $\triangle PQR$ is

A. 5.5. B. 18. C. 23.5. D. 25.

4. Let O be the origin. If the coordinates of the points A and B are (0, 29) and (20, 10) respectively, then the x-coordinate of the orthocentre of $\triangle OAB$ is

A. 9.5. B. 10. C. 14.5. D. 20.

5. Let O be the origin. If the coordinates of the points P and Q are (44, 46) and (67, 0) respectively, then the y-coordinate of the orthocentre of $\triangle OPQ$ is

A. 22. B. 23. C. 33.5. D. 44.

6. Let O be the origin. If the coordinates of the points F, G and H are (0, 50), (60, 50) and (40, 0) respectively, then the y-coordinate of the orthocentre of $\triangle FGH$ is

A. 16. B. 27. C. 34. D. 50.

ANS:

1. (a) 10 units **(b)** 17 units **(c)** $\sqrt{34}$ units **(d)** $\sqrt{65}$ units

2. (a) 2 **(b)** 5 **(c)** $\frac{3}{4}$ **(d)** $\frac{7}{6}$ **4. (a)** 26.6° **(b)** 45°

5. 2 **6. (a)** $\frac{5}{3}$ **(b)** $-\frac{1}{6}$

7. (a) $-\frac{2}{7}$ **(b)** $\frac{4}{3}$ **10. (a)** 7 **(b)** 2

11. (a) (4, 1) **(b)** (3, –1) **(c)** (–3, –4) **(d)** (–7, 6)

12. (8, –9) **13.** –18

14. (a) (6, 1) **(b)** (7, –2) **(c)** (5, –7) **(d)** (–1, 5).

15. (–5, 5). **16.** h= –7. k = 1

17. Yes

18. (a) OA = 20 units, AB= 15 units

(b) 150 square units

20. (2, 0). **21.** (0, 5). **23. (a)** $\sqrt{68}$ units

24. (a) 12 **(b)** 57.9 units

25. (a) (0, 15) **(b)** 144 square units

26. (12, 0)

27. (a) (0, 8) **(b)** 114 square units

28. (a) 8 **(b)** 45° **29. (a)** (1, –7)

30. (a) The coordinates of A are (–10, 0)

The coordinates of B are (0, –5)

(b) 25 square units

31. (a) The coordinates of A are (10, 0)

The coordinates of B are (0, –2)

(b) 26 square units

32. (a) 16 **(b)** 47.9 units

33. (a) PQ =5 units, QR= 5 units

RS = 5 units, PS= 5 units

34. (a) (6, 4).

(b) (i) y= –4x – 6 **(ii)** (–2, 2)

(iii) 68 square units

35. (a) (–1, 0)

36. p = 3, q = 5

37. a = 3, b= –2

38. (a) 2 : 1 **(b)** (14, 0)

39. (a) (2, –4) **(c)** 3 : 4

40. The coordinates of Q are (0, –6).

The coordinates of B are (2, –10).

41. (a) 2 : 1 **(b) (i)** 1 : 2 **(ii)** 4 : 2 : 1

42. (a) (6, 0) **(b)** (12, $\frac{21}{2}$) **(c)** 4 : 9

43. (a) The coordinates of M are (0, 12).

The coordinates of N are (8, 0).

(b) 5 : 4

44. (a) The coordinates of P_1 are ($\frac{a+b}{3}$, $\frac{c}{3}$).

The coordinates of P_2 are ($\frac{a+b}{3}$, $\frac{c}{3}$).

45. C **46.** A **47.** D **48.** A **49.** B

DSE type questions:

1. (a) The coordinates of A' are (2, –4).

The coordinates of B' are (–7, 5).

2. (a) The coordinates of P' are (4, –1).

The coordinates of Q' are (1, –13).

3. (a) The coordinates of C' are (–3, 6).

The coordinates of D' are (5, –2).

DSE type MC

1. B 2. D 3. C 4. A 5. A 6. C

English Version

F.3 Trigonometry

Revision Notes:

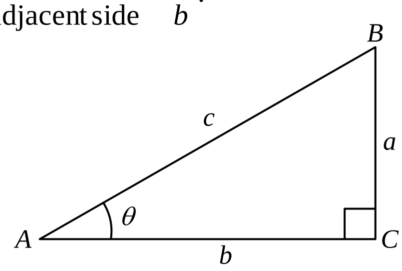
1. Trigonometric Ratios of Acute Angles 銳角的三角比

(a) Refer to the right-angled triangle *ABC* on the right. We have

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{a}{c}$, $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{b}{c}$, $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{a}{b}$.

參看右面的直角三角形 *ABC*，我們得到

$\sin\theta = \dfrac{\text{對邊}}{\text{斜邊}} = \dfrac{a}{c}$，$\cos\theta = \dfrac{\text{鄰邊}}{\text{斜邊}} = \dfrac{b}{c}$，$\tan\theta = \dfrac{\text{對邊}}{\text{鄰邊}} = \dfrac{a}{b}$。



(b) The following table shows the trigonometric ratios of some special angles.

下表顯示一些特殊角的三角比。

Trigonometric ratio \ θ	30°	45°	60°
$\sin\theta$	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$ (or $\dfrac{\sqrt{2}}{2}$)	$\dfrac{\sqrt{3}}{2}$
$\cos\theta$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$ (or $\dfrac{\sqrt{2}}{2}$)	$\dfrac{1}{2}$
$\tan\theta$	$\dfrac{1}{\sqrt{3}}$ (or $\dfrac{\sqrt{3}}{3}$)	1	$\sqrt{3}$

2. Trigonometric Identities 三角恆等式

(a) $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$

(b) $\sin^2\theta + \cos^2\theta \equiv 1$ (or $\sin^2\theta \equiv 1 - \cos^2\theta$ or $\cos^2\theta \equiv 1 - \sin^2\theta$)

(c) $\sin(90° - \theta) \equiv \cos\theta$

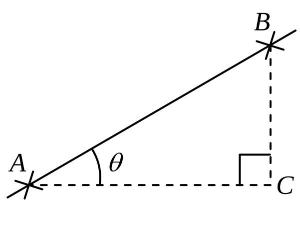
(d) $\cos(90° - \theta) \equiv \sin\theta$

(e) $\tan(90° - \theta) \equiv \dfrac{1}{\tan\theta}$

3. Gradient and Inclination 斜率和傾角

The figure shows an inclined plane *AB*. The ***gradient*** of *AB* $= \dfrac{BC}{AC} = \tan\theta$, where θ is called the ***inclination*** of *AB*.

If the gradient of *AB* is expressed in the form $\dfrac{1}{n}$ or 1 : *n*, then $n = \dfrac{1}{\tan\theta}$.



圖中所示為一個斜面 *AB*。

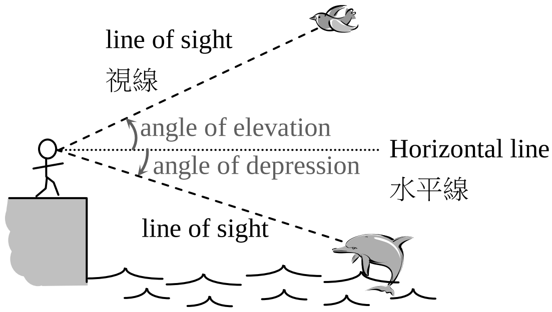
AB 的**斜率** $= \dfrac{BC}{AC} = \tan\theta$，其中 θ 稱為 *AB* 的**傾角**。

若 *AB* 的斜率以 $\dfrac{1}{n}$ 或 1 : *n* 的形式表示，則 $n = \dfrac{1}{\tan\theta}$。

4. Angle of Elevation and Angle of Depression 仰角和俯角

(a) When an observer looks at an object above him, the angle between the line of sight and the horizontal line is called the ***angle of elevation***.

當一個人觀察位於他上方的物件時，視線和水平線之間的夾角稱為**仰角**。



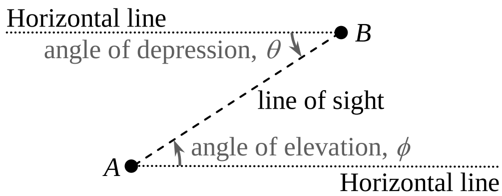
(b) When an observer looks at an object below him, the angle between the line of sight and the horizontal line is called the ***angle of depression***.

當一個人觀察位於他下方的物件時，視線和水平線之間的夾角稱為**俯角**。

(c) Refer to the following figure. The angle of elevation of B from A is equal to the angle of depression of A from B.

參看下圖。由 A 測得 B 的仰角等於由 B 測得 A 的俯角。

i.e. $\theta = \phi$



5. Bearings 方位角

(a) ***Compass bearing*** and ***true bearing*** are two methods of indicating the direction of an object relative to another object.

我們可利用**羅盤方位角**和**真方位角**這兩種方法表示一個物件相對於另一個物件的方位。

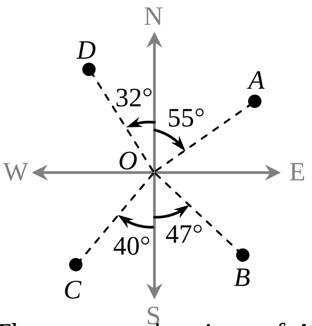
(b) In compass bearing, directions are measured from the north (N) or the south (S) in the form NθE, NθW, SθE and SθW, where $0° < \theta < 90°$.

使用羅盤方位角時，方向由正北或正南開始量度，並以 NθE、NθW、SθE 和 SθW 的形式表示，其中 $0° < \theta < 90°$。

(c) In true bearing, directions are measured from the north in a clockwise direction in a 3-digit form.

使用真方位角時，方向由正北開始按順時針方向量度，並以三位數表示所量得的角度。

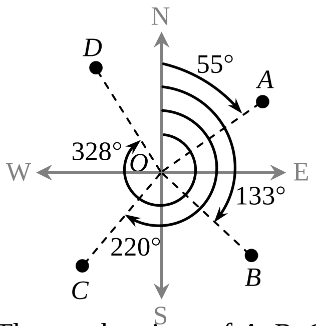
e.g. **(i)**



The compass bearings of A, B, C and D from O are N55°E, S47°E, S40°W and N32°W respectively.

由 O 測得 A、B、C 和 D 的羅盤方位角分別是 N55°E、S47°E、S40°W 和 N32°W。

(ii)

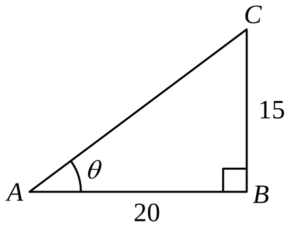


The true bearings of A, B, C and D from O are 055°, 133°, 220° and 328° respectively.

由 O 測得 A、B、C 和 D 的真方位角分別是 055°、133°、220° 和 328°。

Worked examples:

Eg. 3.1 In the figure, find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.
(*Express the answers in fractions.*) （答案以分數表示。）



Solution By Pythagoras' theorem,
利用畢氏定理，

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{20^2 + 15^2}$$
$$= 25$$

$$\sin\theta = \frac{BC}{AC} = \frac{15}{25} = \underline{\underline{\frac{3}{5}}}$$

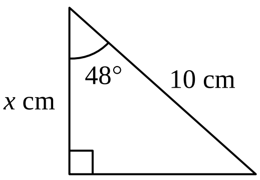
$$\cos\theta = \frac{AB}{AC} = \frac{20}{25} = \underline{\underline{\frac{4}{5}}}$$

$$\tan\theta = \frac{BC}{AB} = \frac{15}{20} = \underline{\underline{\frac{3}{4}}}$$

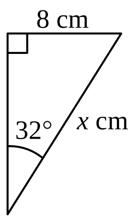
⊂ $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$,
$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$,
$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$.

Eg. 3.2 In each of the following, find x. (*Give the answers correct to 3 significant figures.*)

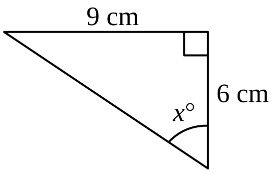
(a)



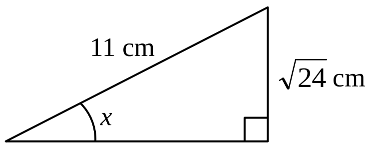
(b)



(c)



(d)



Solution

(a) $\cos 48° = \dfrac{x}{10}$

$x = 10 \cos 48°$

$= \underline{\underline{6.69}}$, *cor. to 3 sig. fig.*

10 [cos] 48 [EXE]
The keying sequences (按鍵次序) here are for calculators of model fx-3650P. Students using other models should refer to the manuals of their own calculators if necessary.

(b) $\sin 32° = \dfrac{8}{x}$

$x = \dfrac{8}{\sin 32°}$

$= \underline{\underline{15.1}}$, *cor. to 3 sig. fig.*

8 [÷] [sin] 32 [EXE]

(c) $\tan x° = \dfrac{9}{6}$

$x = \underline{\underline{56.3}}$, *cor. to 3 sig. fig.*

9 [÷] 6 [EXE] [SHIFT] [tan] ($\tan^{-1}$) [Ans] [EXE]

(d) $\sin x = \dfrac{\sqrt{24}}{11}$

$x = \underline{\underline{26.4°}}$, *cor. to 3 sig. fig.*

[√] 24 [÷] 11 [EXE] [SHIFT] [sin] ($\sin^{-1}$) [Ans] [EXE]
Don't omit the unit of *x*.
不要漏寫 *x* 的單位。

Eg. 3.3

Without using a calculator, find the value of each of the following expressions.
試不用計算機求下列各式的值。

(a) $2 \sin 60° \tan 30°$ **(b)** $\dfrac{\cos 45°}{\tan 45° - \sin 30°}$

(c) $(\tan^2 60° - 1) \cos^2 30°$ **(d)** $\cos 55° \sin 35° + \cos^2 35°$

Solution

(a) $2 \sin 60° \tan 30° = 2 \times \dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{3}}$

$= \underline{\underline{1}}$

sin 60° tan 30° means sin 60° × tan 30°.

(b) $\dfrac{\cos 45°}{\tan 45° - \sin 30°} = \dfrac{\frac{1}{\sqrt{2}}}{1 - \frac{1}{2}}$

$= \dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}$

$\dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \dfrac{1}{\sqrt{2}} \times \dfrac{2}{1}$

$= \underline{\underline{\dfrac{2}{\sqrt{2}}}}$ (or $\sqrt{2}$)

NF $\dfrac{2}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$

(c) $(\tan^2 60° - 1) \cos^2 30° = \left[(\sqrt{3})^2 - 1\right]\left(\dfrac{\sqrt{3}}{2}\right)^2$

$= (3-1)\dfrac{3}{4}$

$= (2)\dfrac{3}{4}$

$= \underline{\underline{\dfrac{3}{2}}}$

$\cos^2 30°$ means cos 30° × cos 30°.

(d) $\cos 55° \sin 35° + \cos^2 35°$

$= \sin(90° - 55°) \sin 35° + \cos^2 35°$ ➲ $\cos\theta = \sin(90° - \theta)$

$= \sin 35° \sin 35° + \cos^2 35°$

$= \sin^2 35° + \cos^2 35°$ ➲ $\sin^2\theta + \cos^2\theta = 1$

$= \underline{\underline{1}}$

Eg. 3.4 Without using a calculator, find the measure of θ in each of the following, where $0° \le \theta < 90°$.
試不用計算機求下列各題中 θ 的大小，其中 $0° \le \theta < 90°$。

(a) $2\sin\theta = \tan 60°$ **(b)** $\tan 2\theta = \dfrac{1}{2\cos 30°}$

Solution **(a)** $2\sin\theta = \tan 60°$

$2\sin\theta = \sqrt{3}$ ➲ $\tan 60° = \sqrt{3}$

$\sin\theta = \dfrac{\sqrt{3}}{2}$

$\theta = \underline{\underline{60°}}$ ➲ $\sin 60° = \dfrac{\sqrt{3}}{2}$

(b) $\tan 2\theta = \dfrac{1}{2\cos 30°}$

$\tan 2\theta = \dfrac{1}{2 \times \dfrac{\sqrt{3}}{2}}$ ➲ $\cos 30° = \dfrac{\sqrt{3}}{2}$

$\tan 2\theta = \dfrac{1}{\sqrt{3}}$

$2\theta = 30°$ ➲ $\tan 30° = \dfrac{1}{\sqrt{3}}$

$\theta = \underline{\underline{15°}}$

Eg. 3.5 Simplify each of the following expressions. 簡化下列各式。

(a) $\dfrac{\cos(90° - \theta)}{\tan\theta}$

(b) $\sin(90° - \theta)\tan(90° - \theta) + \sin\theta$

Solution **(a)** $\dfrac{\cos(90° - \theta)}{\tan\theta} = \dfrac{\sin\theta}{\tan\theta}$ ➲ $\cos(90° - \theta) = \sin\theta$

$= \dfrac{\sin\theta}{\dfrac{\sin\theta}{\cos\theta}}$ ➲ $\dfrac{\sin\theta}{\dfrac{\sin\theta}{\cos\theta}} = \sin\theta \times \dfrac{\cos\theta}{\sin\theta}$

$= \underline{\underline{\cos\theta}}$

(b) $\sin(90° - \theta)\tan(90° - \theta) + \sin\theta = \cos\theta \times \dfrac{1}{\tan\theta} + \sin\theta$ ➲ $\sin(90° - \theta) = \cos\theta$, $\tan(90° - \theta) = \dfrac{1}{\tan\theta}$

$= \cos\theta \times \dfrac{\cos\theta}{\sin\theta} + \sin\theta$ ➲ $\dfrac{1}{\tan\theta} = \dfrac{1}{\dfrac{\sin\theta}{\cos\theta}} = \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\cos^2\theta}{\sin\theta} + \sin\theta$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta}$ ➲ $\cos^2\theta + \sin^2\theta = 1$

$= \underline{\underline{\dfrac{1}{\sin\theta}}}$

Eg. 3.6 Prove each of the following identities. 證明下列各恆等式。

(a) $\dfrac{\cos\theta}{\tan(90^\circ - \theta)} \equiv \sin\theta$

(b) $\tan^2\theta(1 + \sin\theta)(1 - \sin\theta) \equiv \sin^2\theta$

Solution

(a) L.H.S. $= \dfrac{\cos\theta}{\tan(90^\circ - \theta)}$

$= \dfrac{\cos\theta}{\frac{1}{\tan\theta}}$ ⊂ $\tan(90^\circ - \theta) = \dfrac{1}{\tan\theta}$

$= \cos\theta\tan\theta$

$= \cos\theta \times \dfrac{\sin\theta}{\cos\theta}$ ⊂ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \sin\theta$

R.H.S. $= \sin\theta$

∵ L.H.S. = R.H.S.

∴ $\dfrac{\cos\theta}{\tan(90^\circ - \theta)} \equiv \sin\theta$

(b) L.H.S. $= \tan^2\theta(1 + \sin\theta)(1 - \sin\theta)$

$= \tan^2\theta(1 - \sin^2\theta)$ ⊂ Use the identity $(a + b)(a - b) \equiv a^2 - b^2$.

$= \tan^2\theta\cos^2\theta$ ⊂ $1 - \sin^2\theta = \cos^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$

$= \sin^2\theta$

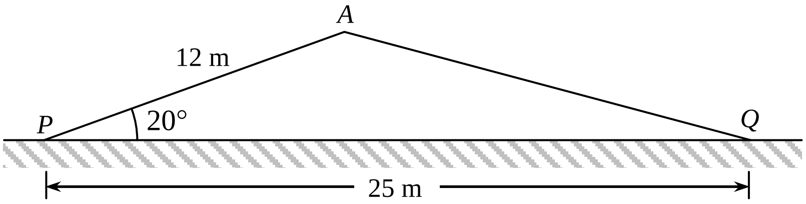
R.H.S. $= \sin^2\theta$

∵ L.H.S. = R.H.S.

∴ $\tan^2\theta(1 + \sin\theta)(1 - \sin\theta) \equiv \sin^2\theta$

Eg. 3.7 In the figure, *AP* and *AQ* represent two straight inclined roads. It is given that $\angle APQ = 20^\circ$, $AP = 12$ m and $PQ = 25$ m.

圖中，*AP* 和 *AQ* 代表兩條筆直的斜路。已知 $\angle APQ = 20^\circ$，$AP = 12$ m 和 $PQ = 25$ m。



(a) Find the gradient of road *AP*. Express the answer in decimal. 求道路 *AP* 的斜率，答案以小數表示。

(b) Find the inclination of road *AQ*. 求道路 *AQ* 的傾角。

(*Give the answers correct to 3 significant figures.*)

Solution

(a) Gradient of road *AP* = tan 20°

= <u>0.364</u>, *cor. to 3 sig. fig.*

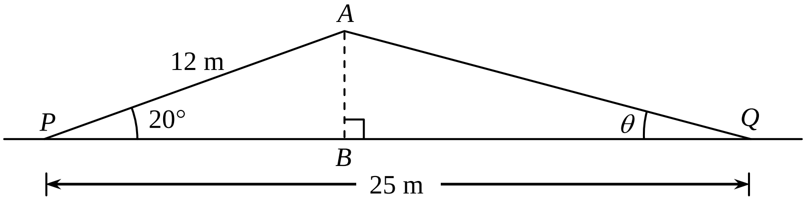
⊂ The gradient of road *AP* can be expressed in the form $1 : \dfrac{1}{\tan 20^\circ}$ or 1 : 2.75 (where 2.75 is correct to 3 significant figures).

(b) Refer to the following figure. Draw a perpendicular line from A to meet PQ at B.

Let the inclination of road AQ be θ.

參看下圖，繪畫一條由 A 至 PQ 的垂線，這垂線與 PQ 相交於 B。

設道路 AQ 的傾角是 θ。



In $\triangle APB$,

$$\sin 20° = \frac{AB}{12\text{ m}}$$

$$AB = 12\sin 20°\text{ m}$$

$$\cos 20° = \frac{PB}{12\text{ m}}$$

$$PB = 12\cos 20°\text{ m}$$

$QB = PQ - PB = (25 - 12\cos 20°)$ m

In $\triangle AQB$,

$$\tan\theta = \frac{AB}{QB} = \frac{12\sin 20°}{25 - 12\cos 20°}$$

$\therefore\ \ \theta = 16.6°$, *cor. to 3 sig. fig.*

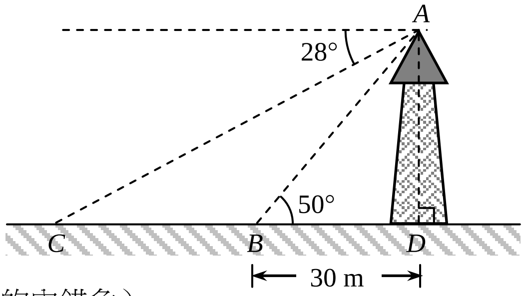
$\therefore$ The inclination of road AQ is 16.6°.

12 sin 20 ÷ (25 − 12 cos 20) EXE SHIFT tan ($\tan^{-1}$) Ans EXE

Eg. 3.8 In the figure, B and C are points on the horizontal ground. The angle of elevation of the top A of a tower from point B is 50°. The angle of depression of point C from A is 28°. The distance between B and the bottom D of the tower is 30 m. B, C and D lie on a straight line. Find the distance between B and C.

圖中，B 點和 C 點位於同一個水平地面上。由 B 點測得一座塔的頂部 A 的仰角是 50°，由 A 測得 C 點的俯角是 28°。B 與塔的底部 D 的距離是 30 m。B、C 和 D 位於同一條直線上。求 B 和 C 的距離。

(*Give the answer correct to 3 significant figures.*)



Solution $\angle ACD = 28°$ (*alt.* ∠*s*, *parallel lines*)（平行線的內錯角）

In $\triangle ABD$,

$$\tan 50° = \frac{AD}{30\text{ m}}$$

$$AD = 30\tan 50°\text{ m}$$

In $\triangle ACD$,

$$\tan\angle ACD = \frac{AD}{CD}$$

$$\tan\angle ACD = \frac{AD}{BC + BD}$$

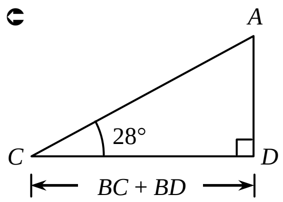
$$BC + BD = \frac{AD}{\tan\angle ACD}$$

$$BC = \frac{AD}{\tan\angle ACD} - BD$$

$$= \left(\frac{30\tan 50°}{\tan 28°} - 30\right)\text{ m}$$

$$= 37.2\text{ m}, \textit{cor. to 3 sig. fig.}$$

$\therefore$ The distance between B and C is 37.2 m.



30 tan 50 ÷ tan 28 − 30 EXE

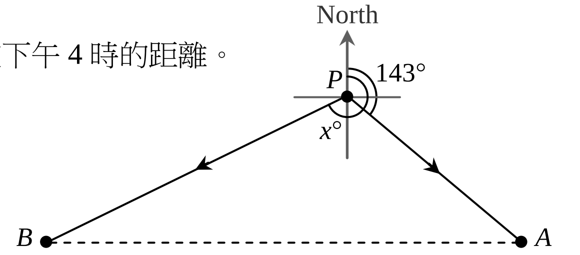
Eg. 3.9 Ships A and B left a pier P at 1 pm. Ship A travelled at a speed of 20 km/h in the direction 143°. Ship B travelled at a speed of 35 km/h in the direction x°, where $180 < x < 270$. If ship A was due east of ship B at 4 pm on the same day, find

船 A 和船 B 在下午 1 時離開碼頭 P。船 A 沿 143° 的方向以速率 20 km/h 航行，船 B 沿 x° 的方向以速率 35 km/h 航行，其中 $180 < x < 270$。若船 A 在同日下午 4 時位於船 B 的正東方，求

(a) x,

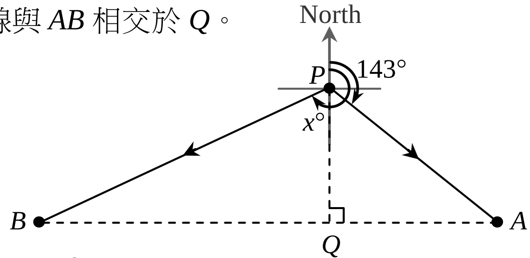
(b) the distance between the two ships at 4 pm. 兩船在下午 4 時的距離。

(*Give the answers correct to 3 significant figures.*)



Solution

(a) Refer to the following figure. Draw a perpendicular line from P to meet AB at Q.
參看下圖，繪畫一條由 P 至 AB 的垂線，這垂線與 AB 相交於 Q。



$\angle APQ = 180° - 143° = 37°$

PA = distance travelled by ship A in 3 hours
船 A 在 3 小時內所航行的距離
= 20 × 3 km
= 60 km

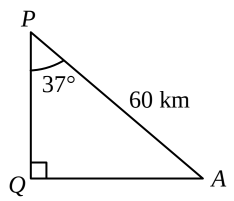
Distance = speed × time taken
距離 = 速率 × 時間

PB = distance travelled by ship B in 3 hours
= 35 × 3 km
= 105 km

In $\triangle APQ$,

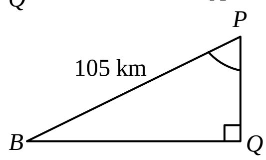
$$\cos\angle APQ = \frac{PQ}{PA}$$

$$PQ = PA\cos\angle APQ = 60\cos 37° \text{ km}$$



In $\triangle BPQ$,

$$\cos\angle BPQ = \frac{PQ}{PB} = \frac{60\cos 37°}{105}$$



$\therefore\ \angle BPQ = 62.847°$, *cor. to 5 sig. fig.*

$\therefore\ x = 180° + \angle BPQ$
$= 180° + 62.847°$
$=$ <u>243°</u>, *cor. to 3 sig. fig.*

To reduce the error (誤差) due to rounding off (四捨五入), 62.847 (correct to 5 significant figures) is used instead of 62.8 (correct to 3 significant figures).

(b) In $\triangle APQ$,

$$\sin\angle APQ = \frac{QA}{PA}$$

$$QA = PA\sin\angle APQ = 60\sin 37° \text{ km}$$

In $\triangle BPQ$,

$$\sin\angle BPQ = \frac{QB}{PB}$$

$$QB = PB\sin\angle BPQ = 105\sin 62.847° \text{ km}$$

$\therefore$ The required distance $= QB + QA$
$= (105\sin 62.847° + 60\sin 37°)$ km
$=$ <u>130 km</u>, *cor. to 3 sig. fig.*

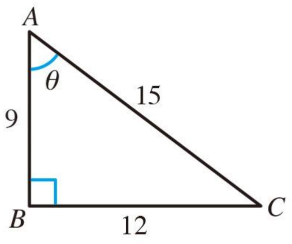
If we use $\angle BPQ = 62.8°$ to find the required distance, we will only get 129 km.

Exercise

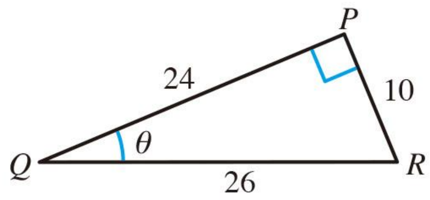
Part 1 Trigonometric Ratios

In each of the following right-angled triangles, find the values of sin θ, cos θ and tan θ. (**1 – 2**)

1.

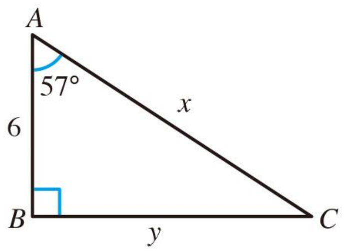


2.

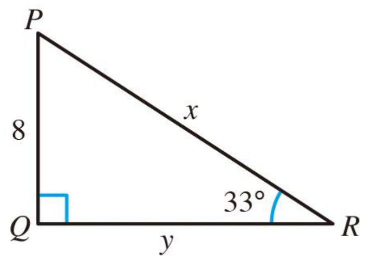


Find the unknowns in each of the following right-angled triangles. (**3 – 10**)

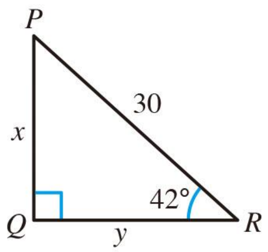
3.



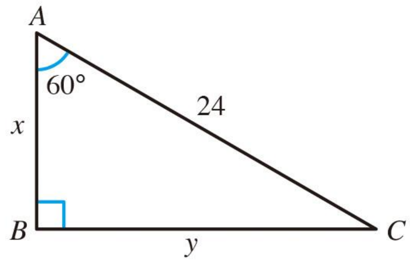
4.



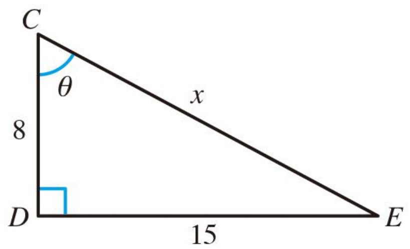
5.



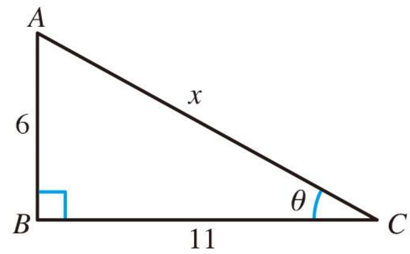
6.



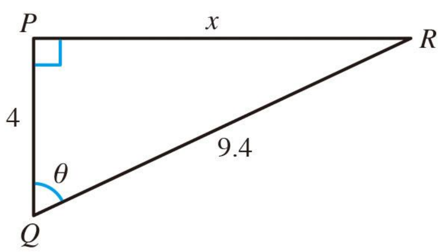
7.



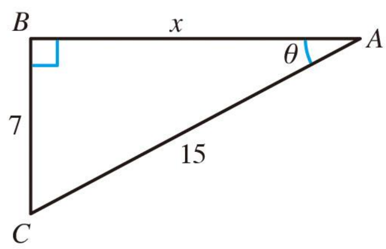
8.



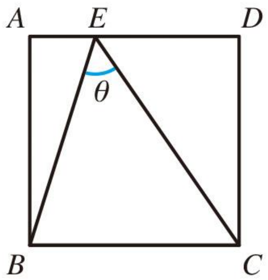
9.



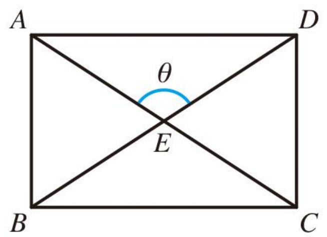
10.



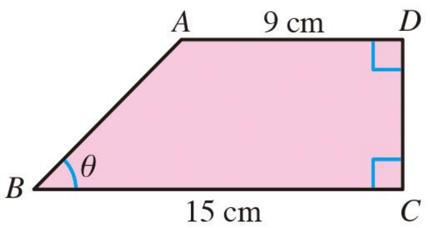
11. In the figure, *ABCD* is a square and *AED* is a straight line. If $AE = 5$ and $AB = 16$, find θ.



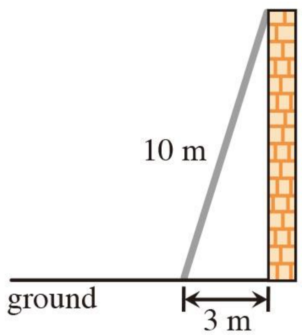
12. In the figure, *ABCD* is a rectangle. The diagonals *AC* and *BD* intersect at *E*. If $BC = 20$ and $CD = 13$, find θ.



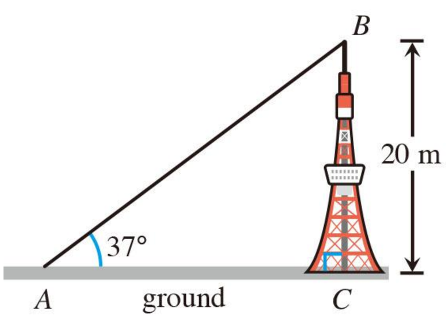
13. If the area of the trapezium *ABCD* shown in the figure is 72 cm^2, find θ.



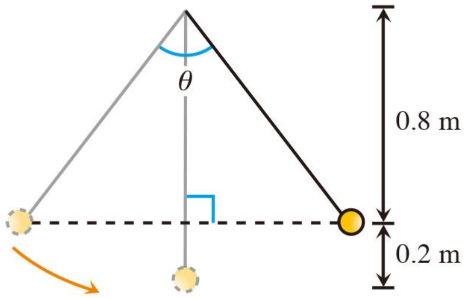
14. In the figure, a ladder of length 10 m leans against a vertical wall. The bottom of the ladder is 3 m away from the wall. Find the angle made by the ladder with the horizontal ground.



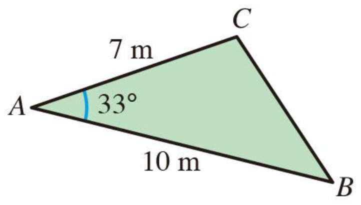
15. In the figure, *BC* is a vertical tower of height 20 m. *A* and *C* are on the same horizontal ground. Given that $\angle BAC = 37°$, find the distance between *A* and *C*.



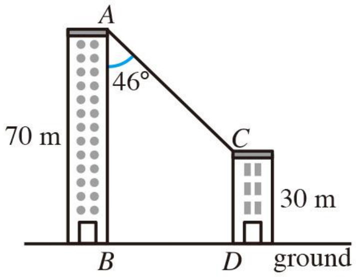
16. An iron ball is attached to the end of a rod. It swings back and forth in a plane. The figure shows different positions of the ball. Find θ.



17. The figure shows a triangular park ABC. Find the area of the park.



18. In the figure, AB and CD are two vertical buildings of heights 70 m and 30 m respectively. B and D are on the same horizontal ground. Given that $\angle BAC = 46°$, find the distance between the two buildings.



Use a calculator to find θ in each of the following. (**19 – 20**)

19. $4 \tan \theta = 7 + \sin 38°$

20. $\tan \theta - 3 \cos 55° = 0$

21. **(a)** Find the value of tan 17°.

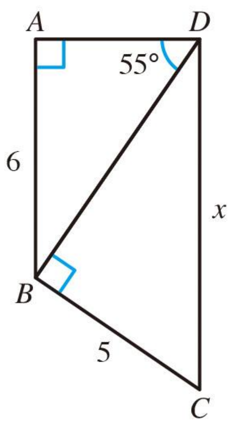
(b) If $\cos \theta = \tan 17°$, find θ.

22. **(a)** Find the value of cos 46°.

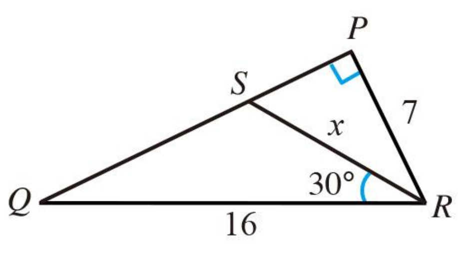
(b) If $2 \sin \theta - \cos 46° = 0$, find θ.

Find the unknown(s) in each of the following figures. (**23 – 26**)

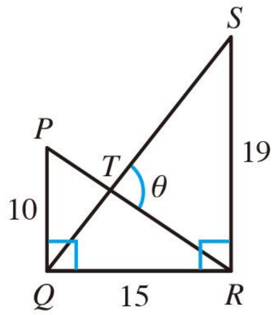
23.



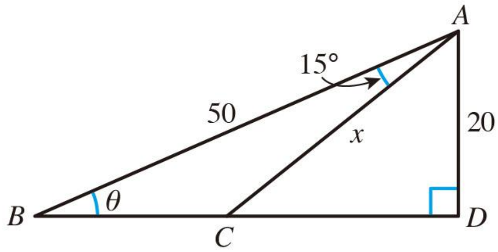
24. PSQ is a straight line.



25. *PR* intersects *QS* at *T*.

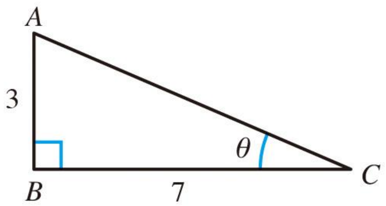


26. *BCD* is a straight line.



Part 2 Trigonometric Relation

1. The figure shows a $\triangle ABC$. $\angle B = 90°$, $BC = 7$ and $AB = 3$. Use Pythagoras' theorem to find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.



2. It is given that $\tan\theta = \frac{4}{9}$. Use Pythagoras' theorem to find the values of $\sin\theta$ and $\cos\theta$.

3. It is given that $\sin\theta = \frac{5}{13}$. Use Pythagoras' theorem to find the values of $\cos\theta$ and $\tan\theta$.

4. It is given that $\cos\theta = 0.125$. Use Pythagoras' theorem to find the values of $\sin\theta$ and $\tan\theta$.

Find the value of each of the following. (**5 – 8**)

5. $(\sin 60° \cos 45°)^2$

6. $\frac{\cos 30°}{\sin 60°} + \sin^2 45°$

7. $\tan^2 45° + \cos^2 60°$

8. $\frac{\cos 60° \tan 30°}{\sin 30° \tan 60°}$

In each of the following, find the value of θ. (**9 – 12**)

9. $2\sin\theta = \sqrt{2}$

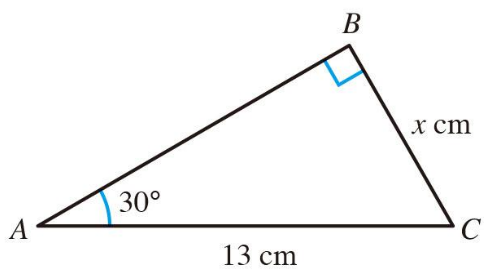
10. $2\cos\theta = 1$

11. $\sqrt{3}\tan\theta = 2\cos 60°$

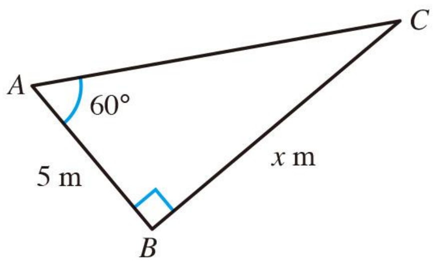
12. $\tan\theta = 2\sin^2 45°$

Find the unknown in each of the following figures. (**13 – 14**)

13.



14.



Simplify each of the following. (**15 – 18**)

15. $\sqrt{1-\sin^2\theta}\cdot\tan\theta$

16. $\dfrac{\cos\theta}{1-\sin^2\theta}\cdot\dfrac{1-\cos^2\theta}{\sin\theta}$

17. $\dfrac{1}{\sin^2(90^\circ-\theta)}-1$

18. $1 - \sin\theta\cos\theta\tan(90^\circ - \theta)$

19. Express $\cos^4\theta + \sin^2\theta\cos^2\theta$ in terms of $\cos\theta$.

20. Express $(1-\cos^2\theta)\left(1+\dfrac{\sin^2\theta}{\cos^2\theta}\right)$ in terms of $\tan\theta$.

Evaluate each of the following. (**21 – 24**)

21. $\dfrac{\cos^2 20^\circ+\cos^2 70^\circ}{\sin^2 42^\circ+\cos^2 42^\circ}$

22. $\cos^2 11^\circ - \sin^2 79^\circ$

23. $\tan 36^\circ \sin^2 54^\circ - \cos 54^\circ \cos 36^\circ$

24. $\tan 25^\circ \sin 65^\circ \sin 25^\circ + \cos^2 25^\circ$

25. Given that $\cos\theta = \dfrac{2}{7}$, find the value of $7\cos(90^\circ - \theta)\tan(90^\circ - \theta)$.

26. Given that $\tan\theta = \dfrac{5}{6}$, find the value of $3\tan\theta - \tan(90^\circ - \theta)$.

In each of the following, find the value of θ. (**27 – 28**)

27. $\cos 66^\circ = \sin\theta$

28. $\tan 36^\circ = \dfrac{1}{\tan(90^\circ-\theta)}$

Prove each of the following identities. (**29 – 32**)

29. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

30. $\sin x\left(\dfrac{1}{\tan x}-\dfrac{1}{\sin x}\right)=\cos x-1$

31. $(\sin x-1)\left[\tan x+\dfrac{1}{\sin(90^\circ-x)}\right]=-\cos x$

32. $\dfrac{1}{1-\sin x}-\dfrac{1}{1+\sin x}=\dfrac{2\tan x}{\cos x}$

33. It is given that $\tan\theta = \dfrac{3}{\sqrt{6}}$. Use Pythagoras' theorem to find the value of $\cos^2\theta\sin\theta$.

34. It is given that $\sin\theta = \dfrac{\sqrt{12}}{5}$. Use Pythagoras' theorem to find the value of $\dfrac{2\tan\theta}{\cos\theta}$.

In each of the following, find the value of θ. (**35 – 40**)

35. $\dfrac{2\cos\theta}{\tan 60^\circ}-\tan 45^\circ=0$

36. $2\cos(\theta + 45^\circ) - 1 = 0$

37. $\tan(50^\circ - \theta) - 1 = 0$

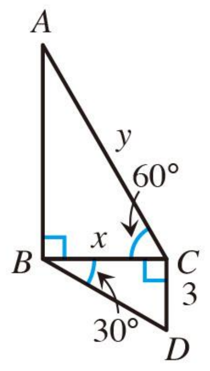
38. $\cos\theta - \sin\theta = 0$

39. $\sqrt{3}\sin\theta=\cos\theta$

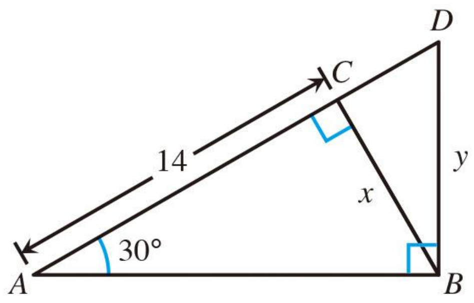
40. $\sin 30^\circ\cos\theta - \cos 30^\circ\sin\theta = 0$

Find the unknowns in each of the following figures. (**41 – 42**)

41.



42. *ACD* is a straight line.



Simplify each of the following. (**43 – 44**)

43. $\dfrac{\cos^2(90^\circ-\theta)}{\sin^2(90^\circ-\theta)}\cdot\dfrac{1}{\tan\theta}$

44. $\dfrac{\sin(90^\circ-\theta)}{\tan(90^\circ-\theta)}-\cos(90^\circ-\theta)$

45. Express $\dfrac{4}{\tan(90^\circ-\theta)}-\dfrac{2\cos(90^\circ-\theta)}{\cos\theta}$ in terms of tan θ.

46. Express $\dfrac{\sin^3\theta+\cos^3\theta\tan\theta}{\sin\theta\cos\theta}$ in terms of cos θ.

47. Given that sin (90° – θ) = $\dfrac{1}{5}$, find the value of $\dfrac{4\tan\theta}{\sin\theta\cos\theta}$.

48. It is given that tan $\alpha = \dfrac{3}{4}$ and sin $\beta = \dfrac{2}{5}$. Find the value of sin α cos β + cos α sin β.

Prove each of the following identities. (**49 – 52**)

49. $\sin\theta+\cos\theta+\tan\theta\sin\theta=\dfrac{1}{\cos\theta}+\cos\theta\tan\theta$

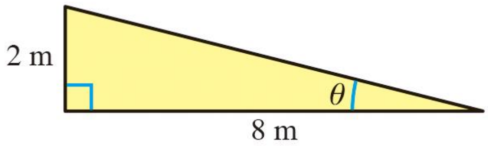
50. $\dfrac{1}{\cos(90^\circ-\theta)}-\sin\theta=\dfrac{\sin(90^\circ-\theta)}{\tan\theta}$

51. $\dfrac{\tan(90^\circ-x)+1}{\tan(90^\circ-x)-1}=\dfrac{1+\tan x}{1-\tan x}$

52. $\dfrac{\cos x}{1-\sin x}=\dfrac{1+\sin x}{\cos x}$

Part 3 Application of Trigonometry

1. The figure shows a ramp with a horizontal distance of 8 m and a vertical distance of 2 m.



(a) Find the gradient of the ramp in fraction.

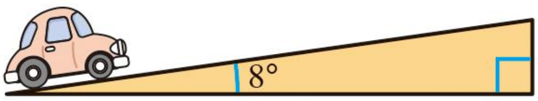
(b) Find the inclination θ of the ramp.

2. The gradient of a straight road is 3 in 13.

(a) What is the inclination of the road?

(b) If the horizontal distance of the road is 50 m, what is the length of the road?

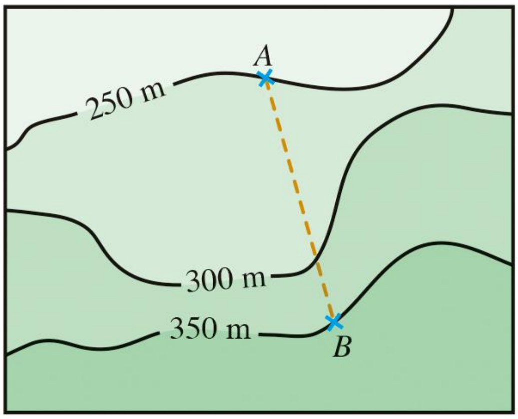
3. In the figure, the inclination of an inclined road is 8°.



(a) By how much does a car rise if the horizontal distance travelled is 2000m?

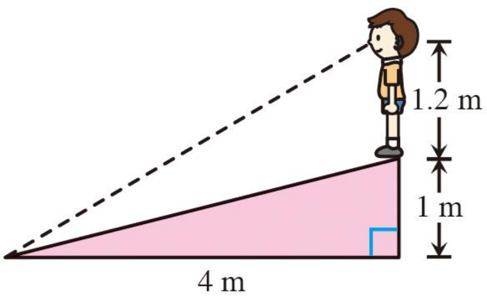
(b) Find the distance travelled by the car along the road when the car has risen 150 m.

4. In the figure, *AB* is a straight road and it is measured as 2.5 cm on the map. If the scale of the map is 1 : 20 000, find the gradient in fraction and the inclination of the straight road *AB*.

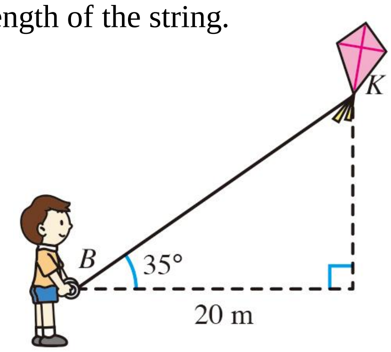


Scale 1 : 20 000

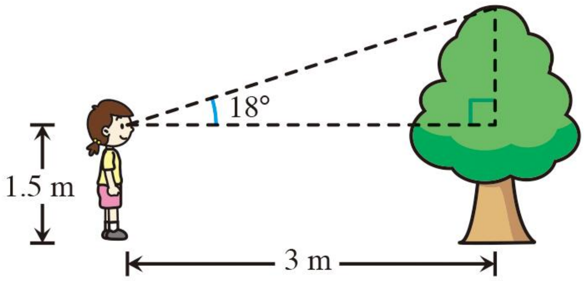
5. In the figure, Jason stands at the top of an inclined stage. If the height of his eye level from the top of the stage is 1.2 m, find the angle of depression of the bottom of the inclined stage from Jason.



6. In the figure, a boy is flying a kite. The horizontal distance between the boy and the kite is 20 m. If the angle of elevation of the kite (K) from the boy (B) is 35°, find the length of the string.
(*Assume that the string is taut*.)

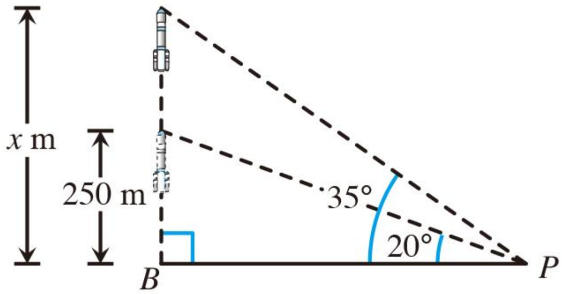


7. In the figure, Susan looks at the top of a tree. The tree is 3 m away from her. Her eye level is 1.5 m above horizontal ground level and the angle of elevation of the top of the tree from her eye is 18°. Find the height of the tree.

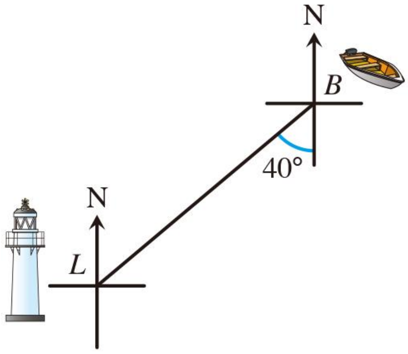


8. In the figure, a rocket is launched vertically from a base B. When it is 250 m above horizontal ground level, the angle of elevation of the rocket from a point P on the ground is 20°. When the rocket is x m above the ground, the angle of elevation from P is 35°.

(a) Find the distance between P and B.
(b) Find the value of x.



9. In the figure, the bearing of a lighthouse L from a boat B is S40°W. Find the compass bearing of the boat from the lighthouse.

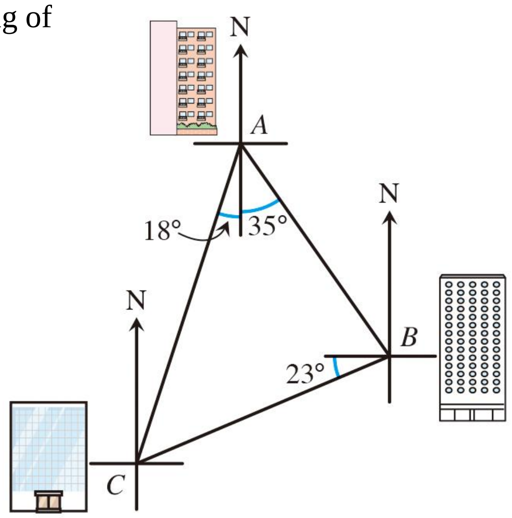


10. The figure on the right shows the locations of three buildings A, B and C. A, B and C are at the same horizontal ground level. Find the true bearing of

(a) A from B,

(b) C from A,

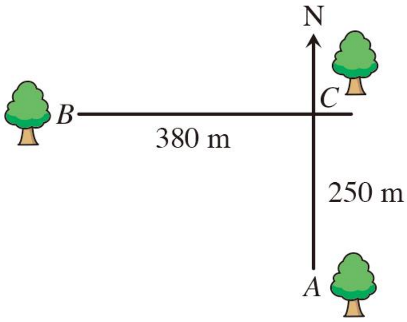
(c) B from C.



11. In the figure, three trees A, B and C are at the same horizontal ground level. A is 250 m due south of C. B is 380 m due west of C.

(a) Find the true bearing of B from A.

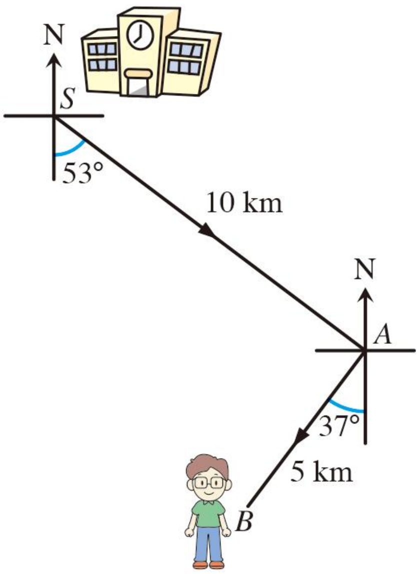
(b) Find the true bearing of A from B.



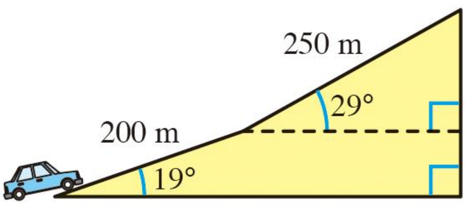
12. In the figure, a student leaves his school (S) and walks in the direction S53°E for 10 km to A. Then he walks in the direction S37°W for 5 km to B.

(a) What is the distance between S and B?

(b) What is the compass bearing of B from S?



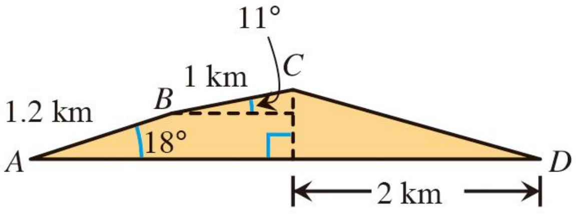
13. In the figure, a car travels 200 m up along a ramp with an inclination of 19° and then 250 m further up along another ramp with an inclination of 29°. What are the horizontal and vertical distances that the car travels?



14. In the figure, Jenny walks up from A to the top C of a hill through B. The inclinations of AB and BC are $18°$ and $11°$ respectively. The length of AB is 1.2 km and that of BC is 1 km. Then she walks down to D which is at the same horizontal level as A. The horizontal distance between C and D is 2 km.

(a) Find the vertical distance between A and C in m.

(b) Find the inclination of CD.

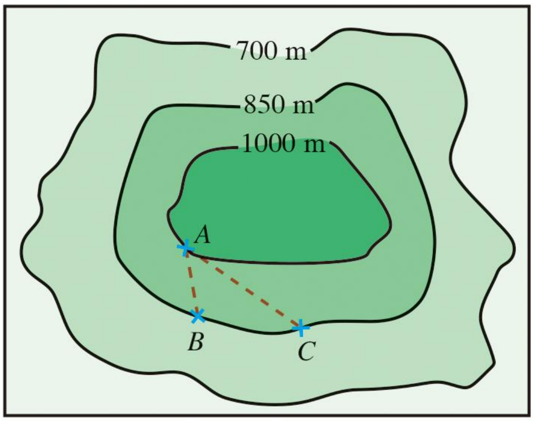


15. The figure shows a contour map with a scale of 1 : 50 000. AB and AC are two straight paths. The lengths of AB and AC are measured on the map as 0.8 cm and 1.6 cm respectively.

(a) Find the inclinations of AB and AC.

(b) Which path, AB or AC, is steeper? Explain your answer.

(c) Find the actual length of the steeper path.

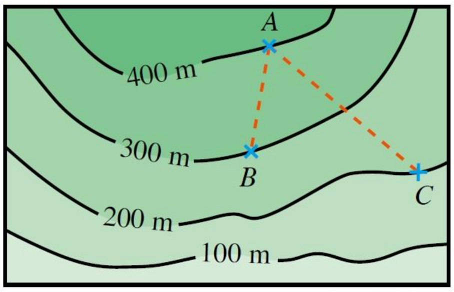


Scale 1 : 50 000

16. The figure shows a contour map with a scale of 1 : n. The lengths of two straight paths AB and AC are measured on the map as 1.2 cm and 2.2 cm respectively. It is given that the inclination of the path AB is $25°$.

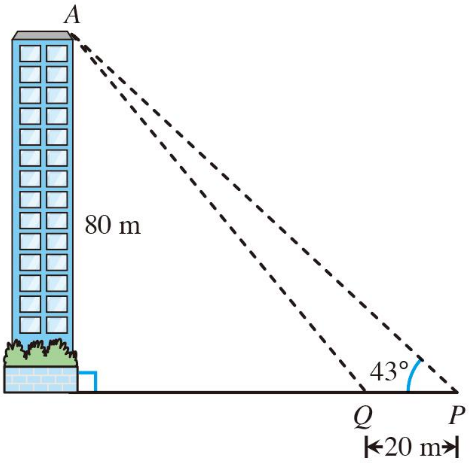
(a) Find the value of n, correct to the nearest thousand.

(b) Using the result of **(a)**, find the inclination of AC.

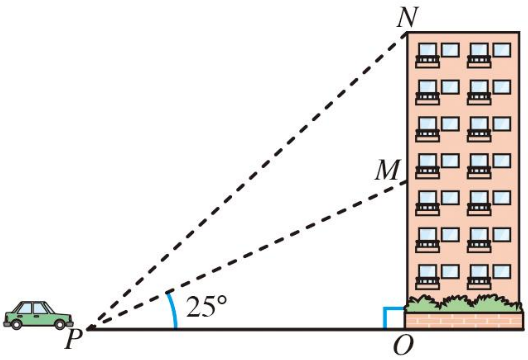


Scale 1 : n

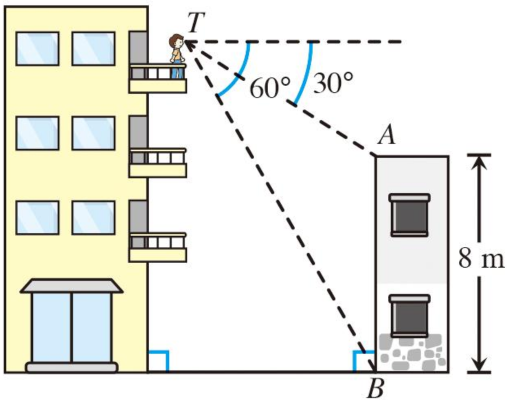
17. In the figure, the height of a building is 80 m. P and Q are two points 20 m apart at horizontal ground level. The angle of elevation of the top A of the building from P is $43°$. It is given that A, P and Q lie on the same vertical plane. Find the angle of elevation of A from Q.



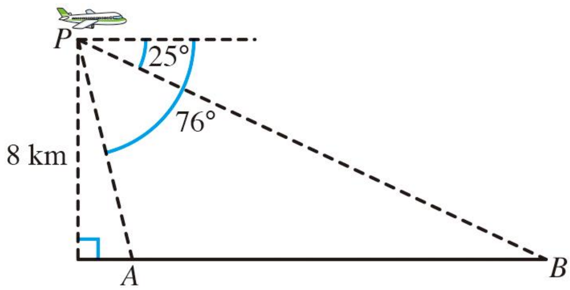
18. The figure shows a building NO. M is a point on the building such that $NO = 2MO$. A car is at a point P at horizontal ground level. It is given that N, M, O and P lie on the same vertical plane. If the angle of elevation of M from P is 25°, find the angle of elevation of N from P.



19. In the figure, a man stands on a balcony in a building. The angles of depression of the top A and the bottom B of a house from the man's eye (T) are 30° and 60° respectively. It is given that T, A and B lie on the same vertical plane. If the height of the house is 8 m, find the height of T above the ground.



20. In the figure, a pilot at P in an aeroplane at a fixed height 8 km above the ground sees two towns A and B. Assume A, B and P lie on the same vertical plane. The angles of depression of the towns A and B from P are 76° and 25° respectively.



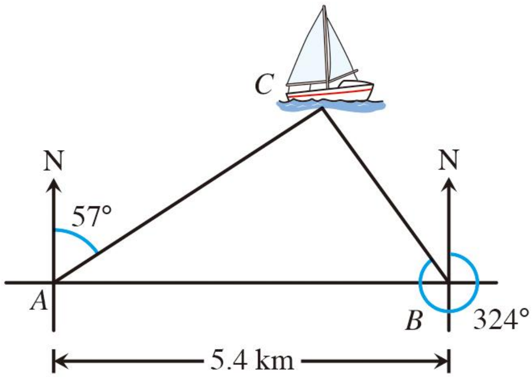
(a) What is the distance between A and B?

(b) If the aeroplane flies horizontally at a constant speed of 200 km/h, find the time (in minutes) required for the aeroplane to reach the point vertically above B.

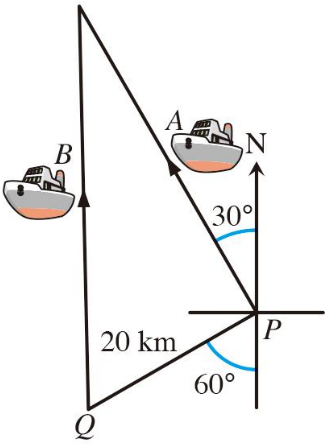
21. In the figure, a radar station *A* is 5.4 km due west of radar station *B*. The bearing of a ship *C* from *A* is 057° while the bearing of *C* from *B* is 324°.

(a) Find the shortest distance between *C* and *AB*.

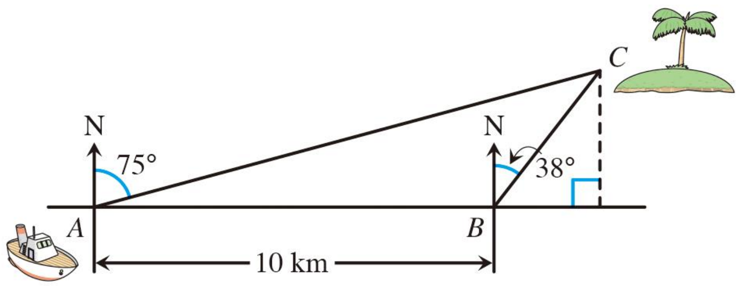
(b) Find the distance between *A* and *C*.



22. In the figure, two boats *A* and *B* sail at constant speeds leaving ports *P* and *Q* respectively at the same time. *A* sails in the direction of N30°W at a speed of 15 km/h. The bearing of *Q* from *P* is S60°W. The distance between *P* and *Q* is 20 km. If boats *A* and *B* meet two hours after leaving the port, find the speed and the sailing direction of boat *B*.



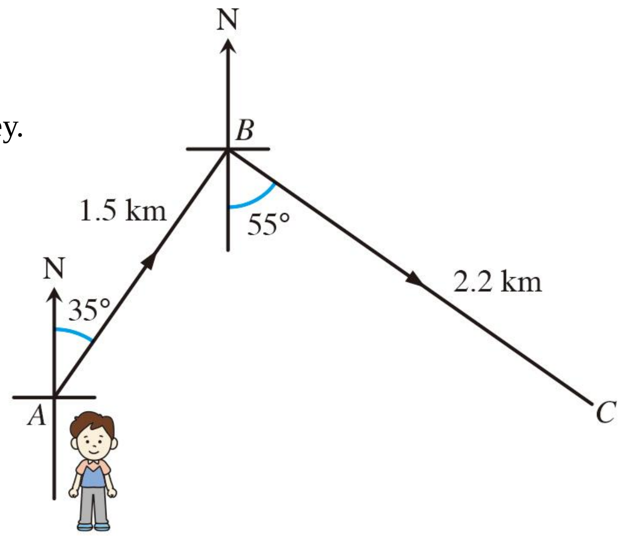
23. In the figure, the bearing of an island *C* from a ship at *A* is N75°E. The ship sails 10 km eastward to *B*. The bearing of *C* from *B* is N38°E. If the ship sails eastward from *B* at a constant speed of 10 km/h, find the time (in minutes) required to reach the point at which the distance between *C* and the ship is the shortest.



24. In the figure, Calvin walks 1.5 km in the direction N35°E to *B* from *A*. He then walks 2.2 km in the direction S55°E to *C* from *B*. Finally, he returns straight to *A* from *C* at a constant speed.

(a) Find the direction of his return route.

(b) If he needs to get back to *A* in 10 minutes, write down a possible speed for his return journey.



25. A car travels 12 km from *P* to *Q*. Then the car travels 15 km to *R* from *Q* in the direction N30°E. The bearing of *Q* from *P* is N40°E.

(a) Find the distance between *P* and *R*.

(b) Find the true bearing of *R* from *P*, correct to the nearest degree.

ANS:

Part 1:

1. $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

2. $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

3. x= 11.0, y=9.24

4. x=14.7, y = 12.3

5. x= 20.1, y= 22.3

6. x= 12, y = 20.8

7. x= 17, θ = 61.9°

8. x= 12.5, θ= 28.6°

9. x= 8.51, θ= 64.8°

10. x= 13.3, θ= 27.8°

11. 51.9°

12. 114°

13. 45°

14. 72.5°.

15. 26.5 m.

16. 73.7°

17. 19.1 m^2

18. 41.4 m

19. 62.3°

20. 59.8°

21. **(a)** 0.306, **(b)**72.2°

22. **(a)** 0.695, **(b)** 20.3°

23. 8.87

24. 8.45

25. 85.4°

26. θ= 23.6°, x= 32.1

Part 2:

1. $\sin\theta = \frac{3}{\sqrt{58}}$, $\cos = \frac{7}{\sqrt{58}}$, $\tan\theta = \frac{3}{7}$

2. $\sin\theta = \frac{4}{\sqrt{97}}$, $\cos\theta = \frac{9}{\sqrt{97}}$

3. $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

4. $\sin\theta = \frac{\sqrt{63}}{8}$, $\tan\theta = \sqrt{63}$

5. $\frac{3}{8}$

6. $\frac{3}{2}$

7. $\frac{5}{4}$

8. $\frac{1}{3}$

9. 45°

10. 60°

11. 30°

12. 45°

13. 6.5

14. $5\sqrt{3}$

15. $\sin\theta$

16. $\tan\theta$

17. $\tan^2\theta$

18. $\sin^2\theta$

19. $\cos^2\theta$

20. $\tan^2\theta$

21. 1

22. 0

23. 0

24. 1

25. 2

26. $\frac{13}{10}$

27. 24°

28. 36°

33. $\frac{6}{5\sqrt{15}}$

34. $\frac{10\sqrt{12}}{13}$ $\left(\text{or } \frac{20\sqrt{3}}{13}\right)$

35. 30°

36. 15°

37. 5°

38. 45°

39. 30°

40. 30°

41. x= $3\sqrt{3}$, y= $6\sqrt{3}$

42. $x = \frac{14}{\sqrt{3}}$, y= $\frac{28}{3}$

43. $\tan\theta$

44. 0

45. $2\tan\theta$

46. $\frac{1}{\cos\theta}$

47. 100

48. $\frac{3\sqrt{21}+8}{25}$

Part 3:

1. (a) $\frac{1}{4}$ **(b)** 14.0°

2. (a) 13.0° **(b)** 51.3 m.

3. (a) 281 m **(b)** 1080 m

4. Gradient of AB= $\frac{1}{5}$

The inclination of AB is 11.3°

5. 28.8°

6. 24.4 m

7. 2.47 m

8. (a) 687 m **(b)** 481

9. N40°E.

10. (a) 325° **(b)** 198° **(c)** 067°

11. (a) 303° **(b)** 123°

12. (a) 11.2 km **(b)** S26.4°E

13. Horizontal distance traveled=408 m,

Vertical distance travelled= 186 m

14. (a) 562 m **(b)** 15.7°

15. (a) Inclination of AB =20.6°

Inclination of AC=10.6°

(b) AB is steeper.

(c) 427 m

16. (a) 18 000 **(b)** 26.8°

17. 50.6°

18. 43.0°

19. 12 m

20. (a) 15.2 km **(b)** 5.15 min

21. (a) 2.38 km **(b)** 4.37 km

22. Speed of boat B = 18.0 km/h, sailing direction of boat B : N3.69°E.

23. 15.9 min

24. (a) N89.3°W
(b) 16 km/h (Or other reasonable answers.)

25. (a) 26.9 km **(b)** 034°

English Version

F.3 Laws of Indices and Polynomials

Revision Notes:

1. Zero Index and Negative Integral Indices 零指數和負整數指數

(a) The zero index of any non-zero number a is defined as 1.

任何非零數 a 的零指數定義為 1。

i.e. $a^0 = 1$, where $a \neq 0$.

(b) The negative integral index of any non-zero number a, say a^{-n}, is defined as $\frac{1}{a^n}$.

任何非零數 a 的負整數指數（例如 a^{-n}）定義為 $\frac{1}{a^n}$ 。

i.e. $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$ and n is a positive integer.

e.g. **(i)** $5^0 = \underline{\underline{1}}$, $(-6)^0 = \underline{\underline{1}}$. **(ii)** $3^{-2} = \frac{1}{3^2} = \underline{\underline{\frac{1}{9}}}$, $(-2)^{-5} = \frac{1}{(-2)^5} = \underline{\underline{-\frac{1}{32}}}$.

2. Laws of Integral Indices 整數指數定律

If m and n are integers, $a \neq 0$ and $b \neq 0$, then

(a) $a^m \times a^n = a^{m+n}$ **(b)** $a^m \div a^n = a^{m-n}$ **(c)** $(a^m)^n = a^{m \times n}$

(d) $(ab)^n = a^n b^n$ **(e)** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

e.g. **(i)** $2^7 \times 2^3 = 2^{7+3} = \underline{\underline{2^{10}}}$ $a^7 \times a^3 = a^{7+3} = \underline{\underline{a^{10}}}$

(ii) $2^7 \div 2^3 = 2^{7-3} = \underline{\underline{2^4}}$ $a^7 \div a^3 = a^{7-3} = \underline{\underline{a^4}}$

(iii) $(2^7)^3 = 2^{7 \times 3} = \underline{\underline{2^{21}}}$ $(a^7)^3 = a^{7 \times 3} = \underline{\underline{a^{21}}}$

(iv) $(2b)^3 = 2^3 b^3 = \underline{\underline{8b^3}}$ $(a^2 b)^3 = (a^2)^3 b^3 = a^{2 \times 3} b^3 = \underline{\underline{a^6 b^3}}$

(v) $\left(\frac{2}{b}\right)^3 = \frac{2^3}{b^3} = \underline{\underline{\frac{8}{b^3}}}$ $\left(\frac{a^2}{b}\right)^3 = \frac{(a^2)^3}{b^3} = \frac{a^{2\times3}}{b^3} = \underline{\underline{\frac{a^6}{b^3}}}$

3. Polynomials 多項式

(a) A ***monomial*** is an algebraic expression which can be **(i)** a number, **(ii)** a variable or **(iii)** the product of a number and one or more variables.

e.g. 5, $-2a$, $4xy^2$ and $\frac{1}{6}x^3$ are monomials. $x + 1$, $\frac{1}{x}$ and $\sqrt{x}$ are not monomials.

單項式是一個代數式，它可以是 **(i)** 一個數、**(ii)** 一個變數或 **(iii)** 一個數與一個或多個變數相乘的積。

例如：5、$-2a$、$4xy^2$ 和 $\frac{1}{6}x^3$ 都是單項式。$x + 1$、$\frac{1}{x}$ 和 $\sqrt{x}$ 不是單項式。

(b) The ***coefficient*** of a monomial is its numerical part.

e.g. The coefficient of $-2a$ is -2. The coefficient of $4xy^2$ is 4.

單項式的數字部分稱為單項式的**係數**。

例如：$-2a$ 的係數是 -2，$4xy^2$ 的係數是 4。

(c) The ***degree*** of a monomial is the sum of the indices of all the variables.

e.g. The degree of $-2a$ is 1. The degree of $4xy^2$ is 3.

單項式的**次數**是所有變數的指數之和。

例如：$-2a$ 的次數是 1，$4xy^2$ 的次數是 3。

(d) A monomial or the sum of monomials is called a ***polynomial***. Each monomial is called a ***term*** of the polynomial. Among all the terms in a polynomial, the one with the highest degree determines the degree of the polynomial.

e.g. The degree of $-3x^2 + x^4 + 7$ is 4. The degree of $x^5y - 3x^2y + y^4$ is 6.

一個單項式或多個單項式之和稱為**多項式**，而每個單項式稱為多項式的**項**。一個多項式的次數由其中次數最高的項而定。

例如：$-3x^2 + x^4 + 7$ 的次數是 4，$x^5y - 3x^2y + y^4$ 的次數是 6。

(e) A polynomial can be written in descending or ascending powers of the variable.

e.g. The terms of the polynomial $x^4 - 3x^2 + 7$ are arranged in descending powers of x.

The terms of the polynomial $7 - 3x^2 + x^4$ are arranged in ascending powers of x.

多項式的各項可按變數的降冪或升冪排列。

例如： 多項式 $x^4 - 3x^2 + 7$ 是按變數 x 的降冪排列寫成，而

多項式 $7 - 3x^2 + x^4$ 則是按變數 x 的升冪排列寫成。

(f) Consider the polynomial $x^4 - 3x^2 + 7$. 考慮多項式 $x^4 - 3x^2 + 7$。

(i)

Variable 變數	**Degree** 次數	**Coefficients of** x 各次冪的係數				**Constant term** 常數項	**Number of terms** 項數
		x^4	x^3	x^2	x		
x	4	1	0	-3	0	7	3

(ii) When $x = 1$, the value of the polynomial $= (1)^4 - 3(1)^2 + 7 = 5$.

當 $x = 1$ 時，多項式的值 $= (1)^4 - 3(1)^2 + 7 = 5$。

4. Addition, Subtraction and Multiplication of Polynomials 多項式的加法、減法和乘法

(a) In performing addition and subtraction of polynomials, we should group the like terms together and then combine the like terms.

進行多項式的加法和減法時，我們應先把同類項組合起來，然後合併同類項。

(b) In performing multiplication of polynomials, we can apply the distributive law of multiplication.

進行多項式的乘法時，我們可以利用乘法分配律。

5. Factorization 因式分解

(a) Expressing a polynomial as a product of its factors is called factorization.

把一個多項式寫成其因式之積，這個過程稱為因式分解。

(b) The following methods can be used to factorize polynomials: 我們可利用以下方法因式分解多項式：

(i) Taking out common factors. 提取公因式

(ii) Grouping terms. 併項法

(iii) Using identities [e.g. $a^2 - b^2 \equiv (a + b)(a - b)$, $a^2 + 2ab + b^2 \equiv (a + b)^2$, $a^2 - 2ab + b^2 \equiv (a - b)^2$,

利用恆等式 $a^3 - b^3 \equiv (a - b)(a^2 + ab + b^2)$, $a^3 + b^3 \equiv (a + b)(a^2 - ab + b^2)$].

(iv) The cross-method. 十字相乘法

Worked examples:

Example 1.1 Find the values of the following expressions without using a calculator and give the answers in integers or fractions. 試不用計算機求下列數式的值，答案以整數或分數表示。

(a) $8^2 \times 5^0$ **(b)** $3^{-2} \times (-2)^3$ **(c)** $5^{-2} \div 5^{-6}$ **(d)** $(7^3)^2 \times 7^{-4}$

(e) $(2^3)^{-2} \div (4^{-1})^3$ **(f)** $\left(-\frac{1}{3}\right)^{-5} \times 9^{-2}$

Solution

(a) $8^2 \times 5^0 = 64 \times 1 = \underline{\underline{64}}$ ⊂ $a^0 = 1$

(b) $3^{-2} \times (-2)^3 = \frac{1}{3^2} \times (-8) = \frac{1}{9} \times (-8) = \underline{\underline{-\frac{8}{9}}}$ ⊂ $a^{-n} = \frac{1}{a^n}$

(c) $5^{-2} \div 5^{-6} = 5^{-2-(-6)} = 5^4 = \underline{\underline{625}}$ ⊂ $a^m \div a^n = a^{m-n}$

(d) $(7^3)^2 \times 7^{-4} = 7^{3 \times 2} \times 7^{-4}$ ⊂ $(a^m)^n = a^{m \times n}$

$= 7^6 \times 7^{-4}$

$= 7^{6+(-4)}$ ⊂ $a^m \times a^n = a^{m+n}$

$= 7^2$

$= \underline{\underline{49}}$

(e) $(2^3)^{-2} \div (4^{-1})^3 = 2^{3 \times (-2)} \div 4^{(-1) \times 3}$ ⊂ $(a^m)^n = a^{m \times n}$

$= 2^{-6} \div 4^{-3}$

$= 2^{-6} \div (2^2)^{-3}$ ⊃ $4 = 2^2$

$= 2^{-6} \div 2^{2 \times (-3)}$

$= 2^{-6} \div 2^{-6}$

$= \underline{\underline{1}}$

(f) $\left(-\frac{1}{3}\right)^{-5} \times 9^{-2} = (-3)^5 \times (3^2)^{-2}$ ⊃ $a^{-n} = \frac{1}{a^n}$, $9 = 3^2$.

$= (-1)^5(3^5) \times 3^{2 \times (-2)}$ ⊃ $(-3)^5 = [(-1)(3)]^5 = (-1)^5(3^5)$

$= -3^5 \times 3^{-4}$

$= -3^{5 + (-4)}$ ⊃ $a^m \times a^n = a^{m+n}$

$= \underline{\underline{-3}}$

Example 1.2 Simplify the following expressions and express the answers with positive indices.
(*All the letters in the expressions represent positive numbers*.)
簡化下列數式，答案以正指數表示。(數式中所有字母都代表正數。)

(a) $(a^{-4}b)^{-1}$ **(b)** $(c^{-3})^2(c^{-1})^{-5}$

(c) $\frac{(x^2y)^3}{x^{-4}y^5}$ **(d)** $(m^{-2}n^3)^{-4}\left(\frac{n^2}{m}\right)^5$

Solution

(a) $(a^{-4}b)^{-1} = a^{-4 \times (-1)}b^{-1}$ ⊃ $(ab)^n = a^nb^n$

$= a^4b^{-1}$

$= \underline{\underline{\frac{a^4}{b}}}$ ⊃ $a^{-n} = \frac{1}{a^n}$

(b) $(c^{-3})^2(c^{-1})^{-5} = c^{-3 \times 2}c^{-1 \times (-5)}$ ⊃ $(a^m)^n = a^{m \times n}$

$= c^{-6}c^5$

$= c^{-6+5}$ ⊃ $a^m \times a^n = a^{m+n}$

$= c^{-1}$

$= \underline{\underline{\frac{1}{c}}}$ ⊃ $a^{-n} = \frac{1}{a^n}$

(c) $\frac{(x^2y)^3}{x^{-4}y^5} = \frac{x^{2\times3}y^3}{x^{-4}y^5}$ ⊃ $(ab)^n = a^nb^n$

$= \frac{x^6y^3}{x^{-4}y^5}$

$= x^{6-(-4)}y^{3-5}$ ⊃ $\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$

$= x^{10}y^{-2}$

$= \underline{\underline{\frac{x^{10}}{y^2}}}$

(d) $(m^{-2}n^3)^{-4}\left(\frac{n^2}{m}\right)^5 = m^{-2\times(-4)}n^{3\times(-4)}\left(\frac{n^{2\times5}}{m^5}\right)$ ⊃ $(ab)^n = a^nb^n$, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$= m^8n^{-12}\left(\frac{n^{10}}{m^5}\right)$

$= m^{8-5}n^{-12+10}$ ⊃ $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$

$= m^3 n^{-2}$

$= \dfrac{m^3}{n^2}$

Example 1.3 Simplify $(2a - b) + (2b - 3a)$. 簡化 $(2a - b) + (2b - 3a)$。

Solution

$(2a - b) + (2b - 3a)$

$= 2a - b + 2b - 3a$ ⊂ Remove (撤去) the brackets (括號).

$= 2a - 3a - b + 2b$ ⊂ Group (組合) the like terms (同類項) and combine (合併) the like terms.

$= \underline{\underline{-a + b}}$

Example 1.4 **(a)** Add $2x + x^2 + 3$ to $3x^2 - x + 1$. 求 $3x^2 - x + 1$ 加上 $2x + x^2 + 3$ 的結果。

(b) Subtract $3x^3 + 5x - 6$ from $2x^3 - x^2 + 4$. 求從 $2x^3 - x^2 + 4$ 減去 $3x^3 + 5x - 6$ 的結果。

Solution

(a) $(3x^2 - x + 1) + (2x + x^2 + 3)$

$= 3x^2 - x + 1 + 2x + x^2 + 3$

$= 3x^2 + x^2 - x + 2x + 1 + 3$ ⊂ Arrange the terms in descending powers (降冪) of x and combine the like terms.

$= \underline{\underline{4x^2 + x + 4}}$

(b) $(2x^3 - x^2 + 4) - (3x^3 + 5x - 6)$

$= 2x^3 - x^2 + 4 - 3x^3 - 5x + 6$ ⊂ Beware of the '+' and '–' signs when brackets are removed.

$= 2x^3 - 3x^3 - x^2 - 5x + 4 + 6$

$= \underline{\underline{-x^3 - x^2 - 5x + 10}}$

Example 1.5 Simplify the following polynomials. 簡化下列多項式。

(a) $6x - 4 + 2x^3 - 2x + 1 - x^2 - x^3$ **(b)** $5 - x^4 + 3x^2 - 8 + 7x^4 - 2x^2 + x$

Solution

(a) $6x - 4 + 2x^3 - 2x + 1 - x^2 - x^3$

$= 2x^3 - x^3 - x^2 + 6x - 2x - 4 + 1$

$= \underline{\underline{x^3 - x^2 + 4x - 3}}$

(b) $5 - x^4 + 3x^2 - 8 + 7x^4 - 2x^2 + x$

$= -x^4 + 7x^4 + 3x^2 - 2x^2 + x + 5 - 8$

$= \underline{\underline{6x^4 + x^2 + x - 3}}$

Example 1.6 Multiply $x - 2$ by $4x^2 + 3$. 求 $x - 2$ 乘以 $4x^2 + 3$ 的結果。

Solution

$(x - 2)(4x^2 + 3)$

$= (x - 2)(4x^2) + (x - 2)(3)$

$= (x)(4x^2) - (2)(4x^2) + (x)(3) - (2)(3)$

$= \underline{\underline{4x^3 - 8x^2 + 3x - 6}}$

⊂ $(a + b)(c + d)$
$= (a + b)c + (a + b)d$
$= ac + bc + ad + bd$

Example 1.7 Factorize the following polynomials by taking out common factors.

利用提取公因式的方法，因式分解下列多項式。

(a) $6x^2 - 3xy$ **(b)** $5x^3 - 10x^2 + 15x$

Solution

(a) $6x^2 - 3xy = (3x)(2x) - (3x)(y)$

$= \underline{\underline{3x(2x - y)}}$ ⊂ Take out the common factor $3x$.

(b) $5x^3 - 10x^2 + 15x = (5x)(x^2) - (5x)(2x) + (5x)(3)$

$= \underline{\underline{5x(x^2 - 2x + 3)}}$ ⊂ Take out the common factor $5x$.

Example 1.8 Factorize the following polynomials by grouping terms. 利用併項法，因式分解下列多項式。

(a) $xy + 1 - x - y$ **(b)** $ab - dc + bc - ad$

Solution

(a) $xy + 1 - x - y = (xy - x) + (1 - y)$

$= x(y - 1) - (y - 1)$

$= \underline{(y - 1)(x - 1)}$

➲ Arrange the terms into two groups.

➲ You can expand $(y - 1)(x - 1)$ to check whether the result is the same as the given polynomial.

(b) $ab - dc + bc - ad = (ab - ad) + (bc - dc)$

$= a(b - d) + c(b - d)$

$= \underline{(b - d)(a + c)}$

Example 1.9 Factorize the following polynomials by using identities.
利用恆等式，因式分解下列多項式。

(a) $25x^2 - y^2$ **(b)** $(2x - 1)^2 - x^2$ **(c)** $x^2 + 16x + 64$

(d) $4x^2 - 4xy + y^2$ **(e)** $27x^3 + y^3$ **(f)** $4x^3 - 32y^3$

Solution

(a) $25x^2 - y^2 = (5x)^2 - y^2$

$= \underline{(5x + y)(5x - y)}$

➲ $a^2 - b^2 \equiv (a + b)(a - b)$
$a = 5x$ and $b = y$.

(b) $(2x - 1)^2 - x^2 = (2x - 1 + x)(2x - 1 - x)$

$= \underline{(3x - 1)(x - 1)}$

➲ $a = 2x - 1$ and $b = x$.

(c) $x^2 + 16x + 64 = x^2 + 2(x)(8) + 8^2$

$= \underline{(x + 8)^2}$

➲ $a^2 + 2ab + b^2 \equiv (a + b)^2$
$a = x$ and $b = 8$.

(d) $4x^2 - 4xy + y^2 = (2x)^2 - 2(2x)(y) + y^2$

$= \underline{(2x - y)^2}$

➲ $a^2 - 2ab + b^2 \equiv (a - b)^2$
$a = 2x$ and $b = y$.

(e) $27x^3 + y^3 = (3x)^3 + y^3$

$= (3x + y)[(3x)^2 - (3x)(y) + y^2]$

$= \underline{(3x + y)(9x^2 - 3xy + y^2)}$

➲ $a^3 + b^3 \equiv (a + b)(a^2 - ab + b^2)$
$a = 3x$ and $b = y$.

(f) $4x^3 - 32y^3 = 4(x^3 - 8y^3)$

$= 4[x^3 - (2y)^3]$

$= 4(x - 2y)[x^2 + (x)(2y) + (2y)^2]$

$= \underline{4(x - 2y)(x^2 + 2xy + 4y^2)}$

➲ $a^3 - b^3 \equiv (a - b)(a^2 + ab + b^2)$
$a = x$ and $b = 2y$.

Example 1.10 Factorize the following polynomials by using the cross-method.
利用十字相乘法，因式分解下列多項式。

(a) $x^2 + 2x - 3$ **(b)** $4x^2 - 9x + 2$ **(c)** $6x^2 - 13x - 5$

Solution

(a) $x^2 + 2x - 3 = \underline{(x - 1)(x + 3)}$

➲
$x \quad -1$
$x \quad +3$
$-x \quad +3x \quad = \quad +2x$

(b) $4x^2 - 9x + 2 = \underline{(4x - 1)(x - 2)}$

➲
$4x \quad -1$
$x \quad -2$
$-x \quad -8x \quad = \quad -9x$

(c) $6x^2 - 13x - 5 = \underline{(2x - 5)(3x + 1)}$

$$\begin{array}{ccc} 2x & \times & -5 \\ 3x & & +1 \\ \hline -15x & +2x & = -13x \end{array}$$

Exercise:

<u>Part 1 Indices</u>

Find the values of the following expressions without using a calculator. **(1 – 7)**

(Leave your answers in fractions if necessary.)

1. **(a)** $\left(\frac{1}{4}\right)^0$ **(b)** $\frac{1}{(-9)^0}$

2. **(a)** $7^0 \times 7$ **(b)** $3 \div (-5)^0$

3. **(a)** 5^{-2} **(b)** $(-6)^{-3}$

4. **(a)** $-\left(\frac{5}{2}\right)^0$ **(b)** $\left(-\frac{1}{8}\right)^{-2}$

5. **(a)** $(-3)^3 \times 9^{-1}$ **(b)** $(-2)^{-3} \times (-3)^{-2}$

6. **(a)** $8^{-2} \div 4^{-2}$ **(b)** $9^{-3} \div (-6)^{-2}$

7. **(a)** $2^0 \times \left(\frac{3}{5}\right)^{-3}$ **(b)** $(-3)^{-3} \div \left(\frac{1}{3}\right)^{-2}$

Simplify the following expressions and express your answers with positive indices. **(8 – 18)**

8. **(a)** $(b^{-2})^3$ **(b)** $(t^0)^{-7}$

9. **(a)** $\left(\frac{1}{x^{-2}}\right)^6$ **(b)** $\left(-\frac{1}{y^3}\right)^{-5}$

10. **(a)** $x^9 \times x^{-4}$ **(b)** $y^{-3} \div y^2$

11. **(a)** $2a^{-1} \times 5a^{-5}$ **(b)** $-3b^{-2} \div (6b^{-7})$

12. **(a)** $(v^{-1})^3 \times v^5$ **(b)** $(u^3)^{-3} \div (u^{-2})^{-2}$

13. **(a)** $\frac{a^3 \times a^0}{a^{-7}}$ **(b)** $\frac{b^3}{b^{-2} \times b^9}$

14. **(a)** $(m^5 n^{-4})^{-3}$ **(b)** $\left(-\frac{e^{-2}}{f^2}\right)^4$

15. **(a)** $(4a^2 b^{-1})^{-2}$ **(b)** $(-7c^{-3} d^6)^3$

16. **(a)** $\left(\frac{u^5}{3t^8}\right)^{-2}$ **(b)** $\left(\frac{-2x^3}{y^{-5}}\right)^{-4}$

17. **(a)** $\frac{x^7 y^0}{x^5 y^{-4}}$ **(b)** $\frac{v^3 s^{-1}}{v^{-2} s^3}$

18. **(a)** $\dfrac{b^2}{(ab^{-2})^4}$ **(b)** $\dfrac{(c^2d^{-1})^{-3}}{c^{-4}d^5}$

Find the values of the following expressions without using a calculator. **(19 – 21)**

(Leave your answers in fractions if necessary.)

19. **(a)** $12^0 - 16^{-1} \times 4$ **(b)** $36 \div 6^0 + 3^{-2} \times 9$

20. **(a)** $-81^2 \times 27^{-4}$ **(b)** $49^{-1} \div 7^{-1} - 32 \times (-4)^{-3}$

21. **(a)** $-4^{-4} \times \left(\dfrac{3^{-1}}{4}\right)^{-3}$ **(b)** $\left(-\dfrac{2}{5}\right)^{-4} \div \left(\dfrac{4}{25}\right)^{-3}$

Simplify the following expressions and express your answers with positive indices. **(22 – 29)**

22. **(a)** $\dfrac{(2a^{-2}b^5)^4}{8}$ **(b)** $\dfrac{(p^{-3}q^4)^0}{(3p^4q^{-3})^{-3}}$

23. **(a)** $(4u^3v^{-5})^2 \times uv^2$ **(b)** $6a^4b^3 \times (-2a^{-2}b^4)^{-3}$

24. **(a)** $x^{-7}y^6 \div (5xy^{-2})^3$ **(b)** $(2m^3n^{-2})^5 \div (-4m^5n^{-4})$

25. **(a)** $(p^{-1}q^{-2})^3 \times (p^{-2}q)^{-2}$ **(b)** $(16r^5s^{-2})^{-1} \div (2rs^3)^{-4}$

26. **(a)** $(2p^{-3}q^4 \times 3p^2q^{-1})^2$ **(b)** $\left(\dfrac{2x^{-4}y^5}{4x^{-2}y^{-1}}\right)^{-3}$

27. **(a)** $6hk^{-2} \times (h^{-3}k^2)^{-2} \div (h^0k^{-5})^3$ **(b)** $(-3a^{-2}b)^{-3} \div (6a^3b^{-1})^{-2} \times \dfrac{a^{-9}}{b^{-4}}$

28. **(a)** $\dfrac{(8mn)^0(m^2n)^4}{(4m^2n^{-7})^{-1}}$ **(b)** $\dfrac{(-h^2k)^{-3}}{(3h^{-1}k^4)^2(-h^7k^{-2})^{-2}}$

29. **(a)** $\dfrac{25}{(a^{-2}b)^4} \times \left(\dfrac{5a^2b^{-3}}{3a^{-5}b^{-2}}\right)^{-2}$ **(b)** $\left(\dfrac{x^{-3}y^{-1}}{2}\right)^{-5}\left(\dfrac{8x^7y^{-2}}{y^{-4}}\right)^{-2}$

Simplify the following expressions, where *n* is a positive integer, and express your answers with positive indices. **(30 – 31)**

30. **(a)** $\dfrac{2^{n+2}}{4 \times (2^n)^{-2}}$ **(b)** $\dfrac{3^{-3} \times 9^{1-n}}{3^{2n}}$

31. **(a)** $2^{n-3} \times 4^{-2n} \times 8^{1-n}$ **(b)** $3^{3n-1} \times 27^{n-1} \div 9^{n-1}$

Part 2 Polynomials

1. Complete the following table.

	Polynomial	Number of terms	Coefficient of x^3	Coefficient of x^2	Coefficient of x	Coefficient of x^3y	Constant term	Degree of Polynomial
(a)	$5x - 7x^2 + 8x^3 - 2$							

(b)	$3x^3y - 4x^2 + 6x$							
(c)	$-x^2 + 9x + 3$							
(d)	$8x - 5x^2 + 4x^3 - x^3y - 7$							

2. Consider the polynomial $3a - 6a^3 + 5a^2 - 9$.

(a) Arrange the terms of the polynomial in descending powers of a.

(b) Arrange the terms of the polynomial in ascending powers of a.

3. Consider the polynomial $-8x^2 + 7x - 2x^5 + 11$.

(a) Arrange the terms of the polynomial in descending powers of x.

(b) Arrange the terms of the polynomial in ascending powers of x.

Simplify each of the following expressions. **(4 – 9)**

4. **(a)** $(5x + 9) + (3x - 7)$ **(b)** $(4 - 3y) + (6y - 11)$

5. **(a)** $(2x + y) - (x + 4y)$ **(b)** $(7y - 2x) - (5x + 8y)$

6. **(a)** $(2x^2 - 6x - 1) + (x^2 + 5x + 2)$ **(b)** $(x^2 + 4x + 7) + (x^2 - 9x - 3)$

7. **(a)** $(-3x^2 + 5x - 8) + (2x^2 - 5x - 4)$ **(b)** $(6x^2 - x - 6) + (-4x^2 - 8x - 1)$

8. **(a)** $(5x^2 + 8x + 9) - (x^2 + x + 3)$ **(b)** $(x^2 - 4x - 3) - (6x^2 + 2x + 5)$

9. **(a)** $(-7x^2 + x + 8) - (x^2 - x + 8)$ **(b)** $(2x^2 - 5x - 7) - (-3x^2 - 6x + 9)$

10. **(a)** Add $4x^2 + 2x + 5$ to $-8x^2 - 3x + 10$.

(b) Subtract $-6x^2 + 2x + 7$ from $x^2 - 9x - 1$.

11. **(a)** Add $-2x^3 - 3x^2 + 8x - 11$ to $x^3 - 7x^2 - 6x + 7$.

(b) Subtract $3x^3 + 5x^2 - x - 9$ from $9x^3 - x^2 + 4x - 2$.

Expand each of the following expressions. **(12 – 19)**

12. **(a)** $6x(x + 3)$ **(b)** $-2x(4x - 7)$

13. **(a)** $-x(8 - 5x)$ **(b)** $3x(6x + x^2)$

14. **(a)** $(7x - 2)(4x)$ **(b)** $(2x^2 + x)(-5x^3)$

15. **(a)** $(x - 9)(x + 5)$ **(b)** $(x + 4)(6 - x)$

16. **(a)** $(2x + 1)(x + 7)$ **(b)** $(5x - 2)(3x - 8)$

17. **(a)** $(5x + 8)(3 – 4x)$ **(b)** $(x^2 + 7)(4x + 1)$

18. **(a)** $(x – 6y)(x – y)$ **(b)** $(2x – y)(x + 5y)$

19. **(a)** $(8x + y)(2y – 5x)$ **(b)** $(4y – 9x)(3x – 7y)$

Simplify each of the following expressions. **(20 – 21)**

20. $(3x – 2)(6x + 7) – 9x$ **21.** $8x^2 + (5x + 4)(9 – 2x)$

Simplify each of the following expressions. **(22 – 25)**

22. $(–x^3 + 2x^2 – 5) + (–3x^2 + x + 4)$

23. $(4x^3 – 6x – 7x^2 + 2) – (4x – 6 – 7x^3 +2x^2)$

24. $(2xy + 4xz + z) + (7xz – 5xy) – (3z +xz – 6xy)$

25. $(4x^2 – 7 + 3x^3) – [(8x^2 + 6x + 1) – (5x^3 + 2x – 9)]$

Expand each of the following expressions. **(26 – 29)**

26. **(a)** $4x(5x^2 + x – 6)$ **(b)** $(7x – 8x^2 + 3)(–x^2)$

27. **(a)** $(7 – 6x)(x^2 + 4x + 2)$ **(b)** $(3x + 1)(9x^3 + 5x – 4)$

28. **(a)** $(3x + 2)(x^2 – 7x + 5)$ **(b)** $(4x – 5)(3x^2 – 6x + 7)$

29. **(a)** $(10 – 5x – 4x^2)(2x + 9)$ **(b)** $(2x^2 – 7x + 4)(5 – 8x^2)$

Simplify each of the following expressions. **(30 – 33)**

30. $(x – 2)(9x + 4) + (3x + 1)(8 – 3x)$

31. $(x + 7)^2 – (4x + 9)(1 – 2x)$

32. $(1 – 6x)(3x^3 + 5 – 2x^2) – 7x(4x + 1)$

33. $(3x – 2)(x + 6)(x – 5) + 17x$

Part 3 Factorization

Factorize each of the following expressions. **(1 – 16)**

1. **(a)** $6p^2 + 24$ **(b)** $3x^2 – x$

2. **(a)** $–ab – 7ac$ **(b)** $m^3n – 4mn^2$

3. **(a)** $10x^2 – 5y + 20z$ **(b)** $2cd + d^4 + 9d$

4. **(a)** $12k^3 + 8k^2 – 2k$ **(b)** $–p^3q^3 – p^2q – 6pq^3$

5. **(a)** $3xz + 4yz + 9x + 12y$ **(b)** $6ad - 2cd + 3ab - bc$

6. **(a)** $5m^2 + 7m - 10mn - 14n$ **(b)** $3xy^2 - 12x^2 + 2y^3 - 8xy$

7. **(a)** $pr + 18q + 3pq + 6r$ **(b)** $-5ac + 4bd - 10ab + 2cd$

8. **(a)** $4xy - 15yz + 20y^2 - 3xz$ **(b)** $7m^3 + 3n^2 + 21mn + m^2n$

9. **(a)** $r^2 - 16$ **(b)** $81 - k^2$

10. **(a)** $-9p^2 + 4$ **(b)** $64c^2 - 25$

11. **(a)** $36x^2 - y^2$ **(b)** $-49m^2 + 100n^2$

12. **(a)** $a^2 + 8a + 16$ **(b)** $k^2 - 14k + 49$

13. **(a)** $9s^2 - 12s + 4$ **(b)** $36b^2 + 60b + 25$

14. **(a)** $x^2 - 16xy + 64y^2$ **(b)** $4m^2 + 36mn + 81n^2$

15. **(a)** $96b^2 - 54a^2$ **(b)** $x^2y^4 - x^4y^2$

16. **(a)** $7c^2 - 28c + 28$ **(b)** $-27p^2 - 72pq - 48q^2$

Factorize each of the following expressions. **(17 – 22)**

17. **(a)** $k^3 + 343$ **(b)** $n^3 - 729$

18. **(a)** $216 - z^3$ **(b)** $125x^3 + 64$

19. **(a)** $a^3 + 27b^3$ **(b)** $512p^3 - q^3$

20. **(a)** $125m^3 + 8n^3$ **(b)** $343x^3 - 27y^3$

21. **(a)** $-6v^3 - 48$ **(b)** $81 - 375k^3$

22. **(a)** $108c^3 + 256d^3$ **(b)** $13k^3r^3 - 104s^3$

Factorize each of the following expressions. **(23 – 30)**

23. **(a)** $k^2 + 6k - 16$ **(b)** $y^2 - 9y + 18$

24. **(a)** $x^2 + 11x + 28$ **(b)** $a^2 - 4a - 45$

25. **(a)** $2q^2 - 7q + 5$ **(b)** $3r^2 + r - 14$

26. **(a)** $4b^2 + 24b + 11$ **(b)** $6n^2 - 13n - 19$

27. **(a)** $10s^2 - 29s + 21$ **(b)** $9z^2 + 9z - 40$

28. **(a)** $8a^2 - 26a - 24$ **(b)** $-30p^2 - 27p - 6$

29. **(a)** $9p^2 + 18pq - 7q^2$ **(b)** $5c^2 + 32cd + 12d^2$

30. **(a)** $15x^2 - 16xy + 4y^2$ **(b)** $28m^2 - 20mn - 48n^2$

Factorize each of the following expressions. **(31 – 40)**

31. **(a)** $6m^2n^4 - 21m^3n$ **(b)** $-2p^4q^3 - 8p^2q^4 - 4p^2q^3r$

32. **(a)** $12a^2 - 20ab - 9ac + 15bc$ **(b)** $10x + 20x^3y - 5x^4 - 40y$

33. **(a)** $4de + 2df + 6d + 2e + f + 3$ **(b)** $2m + 8n - 10 - 3mk - 12nk + 15k$

34. **(a)** $9x^2 - 8yz + 6xy - 12xz + 21x - 28z$ **(b)** $abc - 5b^2c + 2bc^3 - 4a^2b + 20ab^2 - 8abc^2$

35. **(a)** $(7r + 2)^2 - 64$ **(b)** $4x^2 - (6y - 5x)^2$

36. **(a)** $(a + 4b)^2 - (3a - 5b)^2$ **(b)** $(8m - 3n)^2 - (2m + 9n)^2$

37. **(a)** $(6p + 1)^2 - 12(6p + 1) + 36$ **(b)** $81 - 18(7k - 5) + (5 - 7k)^2$

38. **(a)** $9c^2 - 16d^2 - (3c - 4d)^2$ **(b)** $25p^2 + 10p - 49q^2 + 14q$

39. **(a)** $4x^2 + 25y^2 - 6x + 15y - 20xy$ **(b)** $36a^2 + 12ab + 4b^3 + 24ab^2 + b^2$

40. **(a)** $p^2 + 4pq + 4q^2 - 100$ **(b)** $9m^2 - 64n^2 - 42m + 49$

Factorize each of the following expressions. **(41 – 44)**

41. **(a)** $(x + 4)^3 + 216$ **(b)** $27a^3 - (a + 2)^3$

42. **(a)** $24 + 3(m - 5)^3$ **(b)** $5(1 - 6k)^3 - 320$

43. **(a)** $(a + b)^3 - (2b - a)^3$ **(b)** $(3x - y)^3 + (x + 5y)^3$

44. **(a)** $27p^3 - 8q + q^3 - 24p$ **(b)** $a^3 - 6a^2b + 12ab^2 - 8b^3$

Factorize each of the following expressions. **(45 – 50)**

45. **(a)** $36 - 27a + 5a^2$ **(b)** $9x^2 - 4y^2 + 16xy$

46. **(a)** $-4k^2 + 3k + 10$ **(b)** $-18n^2 + 45n - 25$

47. **(a)** $-54r - 27 - 24r^2$ **(b)** $42 - 10p - 12p^2$

48. **(a)** $6x^3 + 18x^2 - 24x$ **(b)** $4a^3b - 30a^2b^2 + 36ab^3$

49. **(a)** $(3m - 1)^2 - 4m$ **(b)** $21 - (2k + 5)(k + 3)$

50. **(a)** $(4p + 5)(p - 2) + (p + 2)^2$ **(b)** $(y + 7)(3y - 4) + (1 - y)(5y + 2) - 6$

51. **(a)** Factorize $36 - 48n + 16n^2$.

(b) Using the result of **(a)**, factorize $2m^2 - 72 + 96n - 32n^2$.

52. **(a)** Factorize $k^3 + 1$.

(b) Using the result of **(a)**, factorize $k^5 - k^3 + k^2 - 1$.

53. **(a)** Factorize $(b - 3a)^2 - 4b^2 + 12ab$.

(b) Using the result of **(a)**, factorize $(a^2 - 3a - 10)^2 - 4(a^2 - 10)^2 + 12a(a^2 - 10)$.

ANS:

Part 1:

1. **(a)** 1 **(b)** 1

2. **(a)** 7 **(b)** 3

3. **(a)** $\frac{1}{25}$ **(b)** $-\frac{1}{216}$

4. **(a)** –1 **(b)** 64

5. **(a)** –3 **(b)** $-\frac{1}{72}$

6. **(a)** $\frac{1}{4}$ **(b)** $\frac{4}{81}$

7. **(a)** $\frac{125}{27}$ **(b)** $-\frac{1}{243}$

8. **(a)** $\frac{1}{b^6}$ **(b)** 1

9. **(a)** x^{12} **(b)** $-y^{15}$

10. **(a)** x^5 **(b)** $\frac{1}{y^5}$

11. **(a)** $\frac{10}{a^6}$ **(b)** $-\frac{b^5}{2}$

12. **(a)** v^2 **(b)** $\frac{1}{u^{13}}$

13. **(a)** a^{10} **(b)** $\frac{1}{b^4}$

14. **(a)** $\frac{n^{12}}{m^{15}}$ **(b)** $\frac{1}{e^8 f^8}$

15. **(a)** $\frac{b^2}{16a^4}$ **(b)** $-\frac{343d^{18}}{c^9}$

16. **(a)** $\frac{9t^{16}}{u^{10}}$ **(b)** $\frac{1}{16x^{12}y^{20}}$

17. **(a)** x^2y^4 **(b)** $\frac{v^5}{s^4}$

18. **(a)** $\frac{b^{10}}{a^4}$ **(b)** $\frac{1}{c^2d^2}$

19. **(a)** $\frac{3}{4}$ **(b)** 37

20. **(a)** $-\frac{1}{81}$ **(b)** $\frac{9}{14}$

21. **(a)** $-\frac{27}{4}$ **(b)** $\frac{4}{25}$

22. **(a)** $\frac{2b^{20}}{a^8}$ **(b)** $\frac{27p^{12}}{q^9}$

23. **(a)** $\frac{16u^7}{v^8}$ **(b)** $-\frac{3a^{10}}{4b^9}$

24. **(a)** $\frac{y^{12}}{125x^{10}}$ **(b)** $-\frac{8m^{10}}{n^6}$

25. **(a)** $\frac{p}{q^8}$ **(b)** $\frac{s^{14}}{r}$

26. **(a)** $\frac{36q^6}{p^2}$ **(b)** $\frac{8x^6}{y^{18}}$

27. **(a)** $6h^7k^9$ **(b)** $-\frac{4a^3}{3b}$

28. **(a)** $\frac{4m^{10}}{n^3}$ **(b)** $-\frac{h^{10}}{9k^{15}}$

29. **(a)** $\frac{9}{a^6b^2}$ **(b)** $\frac{xy}{2}$

30. **(a)** 2^{3n} **(b)** $\frac{1}{3^{4n+1}}$

31. **(a)** $\frac{1}{2^{6n}}$ **(b)** 3^{4n-2}

Part 2:

1.

	Polynomial	Number of terms	Coefficient of				Constant term	Degree of polynomial
			x^3	x^2	x	x^3y		
(a)	$5x - 7x^2 + 8x^3 - 2$	4	8	–7	5	0	–2	3
(b)	$3x^3y - 4x^2 + 6x$	3	0	–4	6	3	0	4
(c)	$-x^2 + 9x + 3$	3	0	–1	9	0	3	2
(d)	$8x - 5x^2 + 4x^3 - x^3y - 7$	5	4	–5	8	–1	–7	4

2. **(a)** $-6a^3 + 5a^2 + 3a - 9$

(b) $-9 + 3a + 5a^2 - 6a^3$

3. **(a)** $-2x^5 - 8x^2 + 7x + 11$

(b) $11 + 7x - 8x^2 - 2x^5$

4. (a) $8x + 2$ **(b)** $3y - 7$

5. (a) $x - 3y$ **(b)** $-y - 7x$

6. (a) $3x^2 - x + 1$ **(b)** $2x^2 - 5x + 4$

7. (a) $-x^2 - 12$ **(b)** $2x^2 - 9x - 7$

8. (a) $4x^2 + 7x + 6$ **(b)** $-5x^2 - 6x - 8$

9. (a) $-8x^2 + 2x$ **(b)** $5x^2 + x - 16$

10. (a) $-4x^2 - x + 15$ **(b)** $7x^2 - 11x - 8$

11. (a) $-x^3 - 10x^2 + 2x - 4$ **(b)** $6x^3 - 6x^2 + 5x + 7$

12. (a) $6x^2 + 18x$ **(b)** $-8x^2 + 14x$

13. (a) $-8x + 5x^2$ **(b)** $18x^2 + 3x^3$

14. (a) $28x^2 - 8x$ **(b)** $-10x^5 - 5x^4$

15. (a) $x^2 - 4x - 45$ **(b)** $-x^2 + 2x + 24$

16. (a) $2x^2 + 15x + 7$ **(b)** $15x^2 - 46x + 16$

17. (a) $-20x^2 - 17x + 24$ **(b)** $4x^3 + x^2 + 28x + 7$

18. (a) $x^2 - 7xy + 6y^2$ **(b)** $2x^2 + 9xy - 5y^2$

19. (a) $-40x^2 + 11xy + 2y^2$ **(b)** $-27x^2 + 75xy - 28y^2$

20. $18x^2 - 14$

21. $-2x^2 + 37x + 36$

22. $-x^3 - x^2 + x - 1$

23. $11x^3 - 9x^2 - 10x + 8$

24. $3xy + 10xz - 2z$

25. $8x^3 - 4x^2 - 4x - 17$

26. (a) $20x^3 + 4x^2 - 24x$ **(b)** $8x^4 - 7x^3 - 3x^2$

27. (a) $-6x^3 - 17x^2 + 16x + 14$ **(b)** $27x^4 + 9x^3 + 15x^2 - 7x - 4$

28. (a) $3x^3 - 19x^2 + x + 10$ **(b)** $12x^3 - 39x^2 + 58x - 35$

29. (a) $-8x^3 - 46x^2 - 25x + 90$ **(b)** $-16x^4 + 56x^3 - 22x^2 - 35x + 20$

30. $7x$

31. $9x^2 + 28x + 40$

32. $-18x^4 + 15x^3 - 30x^2 - 37x + 5$

33. $3x^3 + x^2 - 75x + 60$

Part 3:

1. (a) $6(p^2 + 4)$ **(b)** $x(3x - 1)$

2. (a) $-a(b + 7c)$ **(b)** $mn(m^2 - 4n)$

3. (a) $5(2x^2 - y + 4z)$ **(b)** $d(2c + d^3 + 9)$

4. (a) $2k(6k^2 + 4k - 1)$ **(b)** $-pq(p^2q^2 + p + 6q^2)$

5. (a) $(3x + 4y)(z + 3)$ **(b)** $(3a - c)(2d + b)$

6. (a) $(5m + 7)(m - 2n)$ **(b)** $(y^2 - 4x)(3x + 2y)$

7. (a) $(p + 6)(r + 3q)$ **(b)** $(2b + c)(2d - 5a)$

8. (a) $(4y - 3z)(x + 5y)$ **(b)** $(7m + n)(m^2 + 3n)$

9. (a) $(r + 4)(r - 4)$ **(b)** $(9 + k)(9 - k)$

10. (a) $(2 + 3p)(2 - 3p)$ **(b)** $(8c + 5)(8c - 5)$

11. (a) $(6x + y)(6x - y)$ **(b)** $(10n + 7m)(10n - 7m)$

12. (a) $(a + 4)^2$ **(b)** $(k - 7)^2$

13. (a) $(3s - 2)^2$ **(b)** $(6b + 5)^2$

14. (a) $(x - 8y)^2$ **(b)** $(2m + 9n)^2$

15. (a) $6(4b + 3a)(4b - 3a)$ **(b)** $x^2y^2(y + x)(y - x)$

16. (a) $7(c - 2)^2$ **(b)** $-3(3p + 4q)^2$

17. (a) $(k + 7)(k^2 - 7k + 49)$ **(b)** $(n - 9)(n^2 + 9n + 81)$

18. (a) $(6 - z)(36 + 6z + z^2)$ **(b)** $(5x + 4)(25x^2 - 20x + 16)$

19. (a) $(a + 3b)(a^2 - 3ab + 9b^2)$ **(b)** $(8p - q)(64p^2 + 8pq + q^2)$

20. (a) $(5m + 2n)(25m^2 - 10mn + 4n^2)$ **(b)** $(7x - 3y)(49x^2 + 21xy + 9y^2)$

21. (a) $-6(v + 2)(v^2 - 2v + 4)$ **(b)** $3(3 - 5k)(9 + 15k + 25k^2)$

22. (a) $4(3c + 4d)(9c^2 - 12cd + 16d^2)$ **(b)** $13(kr - 2s)(k^2r^2 + 2krs +$

$4s^2)$

23. (a) $(k-2)(k+8)$ **(b)** $(y-6)(y-3)$

24. (a) $(x+4)(x+7)$ **(b)** $(a-9)(a+5)$

25. (a) $(q-1)(2q-5)$ **(b)** $(r-2)(3r+7)$

26. (a) $(2b+1)(2b+11)$ **(b)** $(n+1)(6n-19)$

27. (a) $(2s-3)(5s-7)$ **(b)** $(3z-5)(3z+8)$

28. (a) $2(a-4)(4a+3)$ **(b)** $-3(2p+1)(5p+2)$

29. (a) $(3p-q)(3p+7q)$ **(b)** $(c+4d)(5c+3d)$

30. (a) $(3x-2y)(5x-2y)$ **(b)** $4(m+n)(7m-12n)$

31. (a) $3m^2n(2n^3-7m)$ **(b)** $-2p^2q^3(p^2+4q+2r)$

32. (a) $(3a-5b)(4a-3c)$ **(b)** $5(2-x^3)(x-4y)$

33. (a) $(2d+1)(2e+f+3)$ **(b)** $(2-3k)(m+4n-5)$

34. (a) $(3x-4z)(3x+2y+7)$ **(b)** $b(c-4a)(a-5b+2c^2)$

35. (a) $(7r+10)(7r-6)$ **(b)** $3(2y-x)(7x-6y)$

36. (a) $(4a-b)(9b-2a)$ **(b)** $12(5m+3n)(m-2n)$

37. (a) $(6p-5)^2$ **(b)** $49(2-k)^2$

38. (a) $8d(3c-4d)$ **(b)** $(5p+7q)(5p-7q+2)$

39. (a) $(2x-5y)(2x-5y-3)$ **(b)** $(6a+b)(6a+b+4b^2)$

40. (a) $(p+2q+10)(p+2q-10)$ **(b)** $(3m+8n-7)(3m-8n-7)$

41. (a) $(x+10)(x^2+2x+28)$ **(b)** $2(a-1)(13a^2+10a+4)$

42. (a) $3(m-3)(m^2-12m+39)$ **(b)** $-45(2k+1)(12k^2-12k+7)$

43. (a) $(2a-b)(a^2-ab+7b^2)$ **(b)** $4(x+y)(7x^2-10xy+31y^2)$

44. (a) $(3p+q)(9p^2-3pq+q^2-8)$ **(b)** $(a-2b)^3$

45. (a) $(a-3)(5a-12)$ **(b)** $(x+2y)(9x-2y)$

46. (a) $-(k-2)(4k+5)$ **(b)** $-(3n-5)(6n-5)$

47. (a) $-3(2r+3)(4r+3)$ **(b)** $-2(2p-3)(3p+7)$

48. (a) $6x(x-1)(x+4)$ **(b)** $2ab(2a-3b)(a-6b)$

49. (a) $(m-1)(9m-1)$ **(b)** $-(k+6)(2k-1)$

50. (a) $(p-1)(5p+6)$ **(b)** $-2(y-8)(y-2)$

51. (a) $4(2n-3)^2$ **(b)** $2(m+4n-6)(m-4n+6)$

52. (a) $(k+1)(k^2-k+1)$ **(b)** $(k-1)(k+1)^2(k^2-k+1)$

53. (a) $-3(b-3a)(a+b)$ **(b)** $-3(a-5)(a+2)(a^2+a-10)$